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G. N. WATSON 1934

The California Institute of Technology Electric Analog Computer

Introduction. The California Institute of Technology Analog Computer has been developed to provide a general-purpose device having as wide a field of application as is considered practicable using the electric analog principle. The vast majority of problems in the general field of engineering analysis usually require no more than a one per cent accuracy in their solutions. With this lenient accuracy requirement, many simplifications can be made in the computer and its operation. Electrical resistance, inductance, capacitance, and transformer circuits can be used to simulate the linear terms of both algebraic and differential equations.[1–7] This choice of circuits is in contrast to other electric analog computers, such as REAC (Reeves Electronic Analog Computer), which employ electronic amplifiers for all integration and differentiation operations. The electronic analog computers can provide higher accuracy and can be used as simulators on a one-to-one time base, but they are more restricted in their range of application than computers of the type discussed here.

With electric analog computers only one independent variable can be continuously represented. However, by the use of finite-difference methods, such as those employed with digital computers, more independent variables can be handled. The computer at the California Institute of Technology is designed to handle linear partial differential equations with up to three independent variables and non-linear partial differential equations with two independent variables. Two basic circuit techniques are employed. In the first, a single independent variable is represented continuously as time on the computer.[2, 15, 18] In the other, discrete values of several independent variables are represented in a form suitable for the application of finite-difference methods; in this case, the analogous circuit becomes a steady-state a-c or d-c mesh.[15] Both techniques are applicable to both ordinary and partial differential equations. More complex electronic equipment is required in the application of the first principle. It involves devices to generate suitable functions of the independent and dependent variables, multipliers for multiplying any two variables together, devices for simulating arbitrary functions of a dependent variable, and amplifiers to provide the proper power output loads from these devices and to produce negative impedance terms and the unilateral or unsymmetrical terms of a matrix or servo-system.

Description of the Computer. In July 1947 the Institute's computer was placed in active service although it was only partially completed.[1–3] Since then it has been in full-time use for the solution of commercial engineering problems and for Institute research. The computer is best described by referring to Fig. 1 and Table[21] which show its general layout and list the component parts. Its design embodies 110 separate sets of precision inductors, resistors, and capacitors, and 25 special transformers. These circuit elements are used to set up circuits for solving ordinary or partial differential equations. The inductors have a "Q" of better than one hundred under all conditions of operation, and every element can be set to within one per cent.

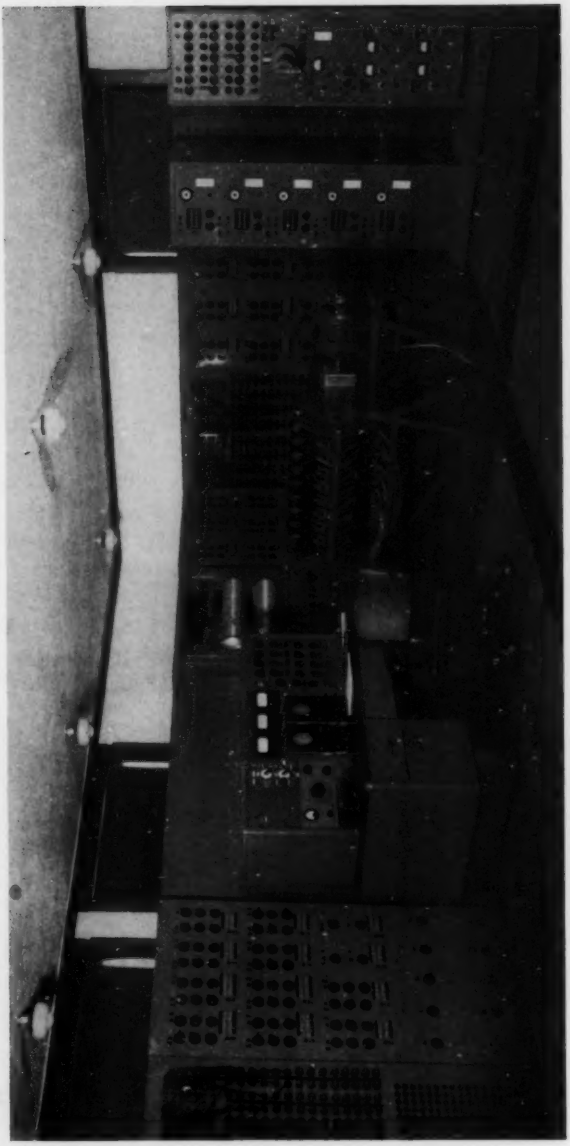


FIG. 1. General View of the California Institute of Technology Computer.

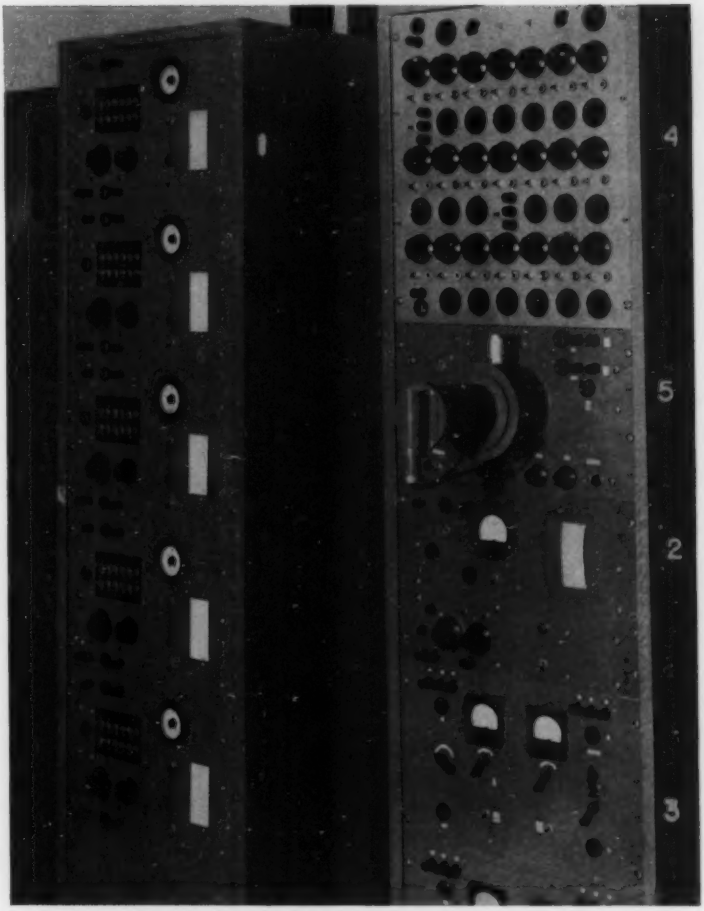


FIG. 2. Photograph of Amplifiers and Arbitrary Function Generators.

1. Negative gain servo-amplifiers.
2. Positive gain (d-c) amplifiers.
3. Voltage and current limiters.
4. Dependent variable functions generator—Type I.
5. Dependent variable functions generator—Type II.

There are, at present, 32 separate forcing functions for introducing (as known voltages, currents, or charge) the known functions of the independent variable. These may be steady-state, variable-frequency, sinusoidal functions; square wave transient functions; or perfectly arbitrary functions of time. As listed in the Table, 20 amplifiers are now available to represent negative impedance terms and unilateral terms such as the unsymmetrical terms of a matrix. For simulating the non-linear terms of an equation, ten multi-

pliers are provided to multiply any two variables, and 11 arbitrary function elements are provided to form arbitrary functions of a dependent variable (see Fig. 2). Five current- and voltage-limiting devices are available for special non-linearities such as missile rudder stops or similar abrupt changes on a dependent variable. The dependent variables that constitute the desired solutions are in all cases simulated by circuit voltages, currents, or charges. For transient solutions, they are recorded or viewed on specially adapted cathode-ray oscilloscopes, while for steady-state or algebraic solutions, they are measured with vacuum-tube or dynamometer-type meters.

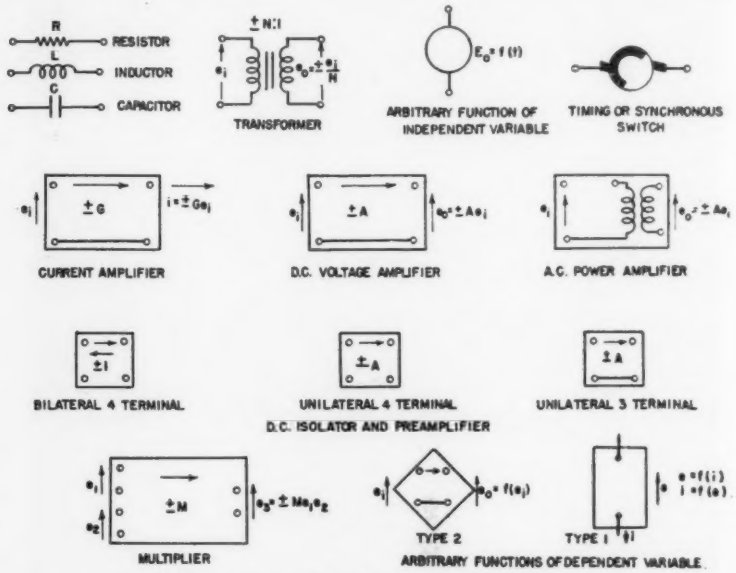
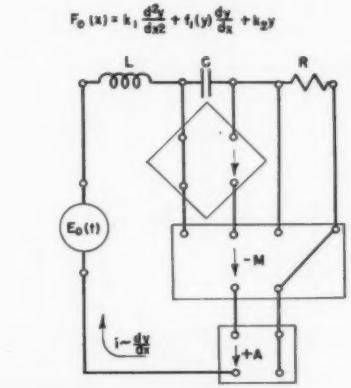


FIG. 3. Symbolization of Basic Computer Elements.

The synchronous switches are important elements of the computer used to switch in the forcing functions, to remove energy from the circuits in order to restore them to their proper initial condition, and to make other sudden changes in circuit conditions. Thirty-six of these switches are now available. They are required only for transient solutions, which may be obtained in either of two ways. One way is to apply the transient just once and record it photographically or with some other memory device such as magnetic tape. The other way is to apply the transient cyclically (usually ten times a second) so that the solution appears as a steady image on a cathode-ray oscilloscope. In both techniques, each switch can be very accurately timed for opening and closing operations. Other important features of this analog computer are the plug-board arrangement and the metering circuits which permit quick connection and rapid metering of all solutions.[6] Proper voltages or currents can be readily inserted at the boundaries or mesh points of circuits which simulate difference-equation approximations to partial differential equations.

A special effort was made to design all elements of the computer so that additions can be readily made if sufficiently important problems arise that require greater facilities.

Examples of Computer Techniques. The required specifications for the analog elements are best considered by first illustrating their application to specific simple problems and describing the analog techniques employed. The basic analog elements are listed in Fig. 3 in terms of the symbols employed for circuit layouts. Typical analogous circuits for both ordinary and partial differential equations are given in Figs. 4 to 7. Figs. 4 to 6 describe



ANALOGOUS CIRCUIT EQUATION:

$$E_0(t) = L\frac{d^2q}{dt^2} + \frac{AMR}{C}F_1(q)\cdot\frac{dq}{dt} + \frac{q}{C}$$

$$\left(R\frac{dq}{dt}\text{ NEGLIGIBLE}\right)$$

NUMERICAL CONVERSION FORMULA:

$$y = a\frac{F_0}{E_0}q\,;\quad L = \frac{a}{n^2}k_1\,;\quad \frac{1}{C} = ak_2\,;\quad \frac{AMR}{C} = F_1(q) = \frac{a}{n}f_1(q)$$

WHERE a = IMPEDANCE BASE CHANGE FACTOR

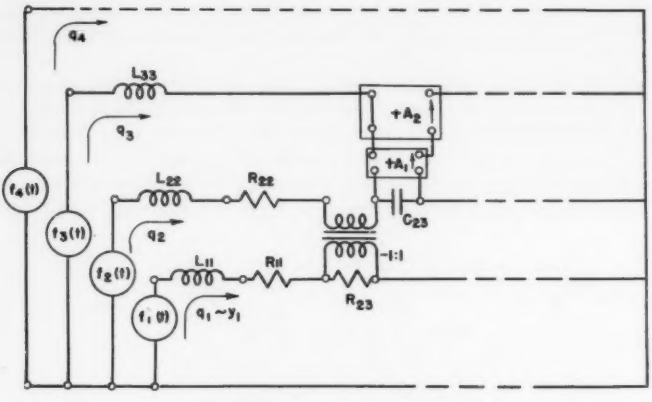
AND n = INDEPENDENT VARIABLE CONVERSION FACTOR

(ONE UNIT OF x IN ITS SYSTEM OF UNITS EQUALS $\frac{1}{n}$ UNITS OF t IN SECONDS.)

FIG. 4. Illustrative Analogy for Ordinary Differential Equation.

techniques in which time continuously represents one independent variable on the computer. Fig. 7 illustrates the solution of partial differential equations with a steady-state network, a very useful iterative method of solving many non-linear partial differential equations.[15]

Fig. 4 illustrates the basic technique of forming an equation by one or both of KIRCHHOFF's circuit laws, the use of inductors and resistors for linear differential terms, and the use of both the multiplier and non-linear function devices for non-linear terms. Fig. 5 shows the use of transformers for mutual bilateral terms and the isolator and amplifier for unilateral or non-symmetrical terms of a matrix of simultaneous equations.



$$f_1(t) = K_{11}\frac{d^2y_1}{dt^2} + k_{11}\frac{dy_1}{dt} + k_{12}\frac{dy_2}{dt} + \text{----}$$

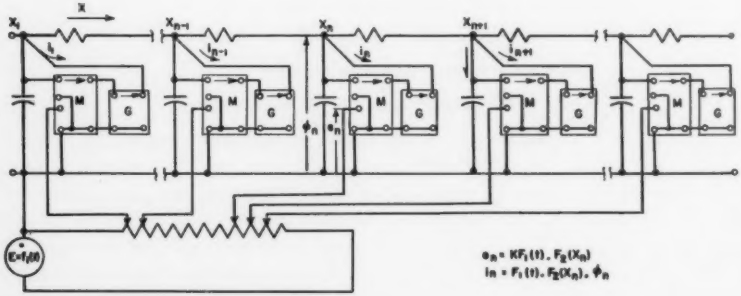
$$f_2(t) = K_{22}\frac{d^2y_2}{dt^2} + k_{22}\frac{dy_2}{dt} + k_{12}\frac{dy_1}{dt} + \text{----}$$

$$f_3(t) = K_{33}\frac{d^2y_3}{dt^2} + k_{23}y_2 \quad \text{----}$$

$$f_4(t) = \text{----}$$

FIG. 5. Illustrative Analogy for Simultaneous Equations.

In Fig. 6, a partial differential equation is considered which requires multipliers and represents one of the independent variables continuously as time on the computer. This technique has limited application to non-linear partial differential equations, since too many special electronic elements would be required for general application.



FINITE DIFFERENCE EQUATION:

$$\frac{\phi_{n+1}-\phi_n}{\Delta X \Delta X_n} - \frac{\phi_n - \phi_{n-1}}{\Delta X \Delta X_{n-1}} = k\frac{\partial \phi_n}{\partial t} + F_1(t)\cdot F_2(X_n)\cdot\phi_n$$

COMPLETE TIME ANALOGY FOR PARTIAL DIFFERENTIAL EQUATION OF HEAT TRANSFER.

$$\frac{\partial^2\phi}{\partial X^2} = k\frac{\partial \phi}{\partial t} + F_1(t)\cdot F_2(X)\cdot\phi$$

FIG. 6. Complete Time Analogy for Partial Differential Equation of Heat Transfer.

The general technique illustrated in Fig. 7 for non-linear partial differential equations has so far been found to have a surprisingly wide field of application to equations containing terms up to the fourth order of differentiation and to pairs of simultaneous equations. Since circuit element settings and inserted mesh currents can be changed rapidly, iterative methods of solution can be set up in a very short time.

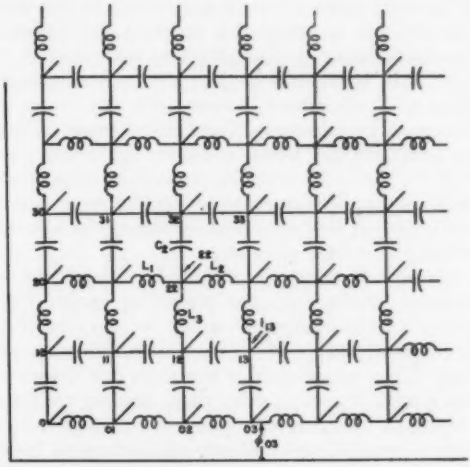

FINITE DIFFERENCE EQUATION AT JUNCTION 22:
BY SUMMATION OF CURRENTS AT JUNCTION 22:
ANALOGOUS CIRCUIT FOR NON-LINEAR PARTIAL DIFFERENTIAL EQUATIONS
OF THE FORM
WITH TIME
SIMULATED AS A SPACE VARIABLE.

FIG. 7.

As illustrated in the examples, RLC circuit elements are usually employed for all linear self-impedance terms. They are also employed for bilateral mutual terms, with junction points between the meshes simulating the simultaneous equations, and for transfer functions. Special transformers[5] are used, wherever possible, for bilateral mutuals requiring circuit isolation and, in conjunction with amplifiers, to provide a four-terminal unilateral element. The RLC circuit elements set the speed of computation. In order to have the elements behave as near-ideal resistors, inductors, and capacitors, the natural frequencies of the analog circuits must lie within certain limits. The normal operating frequency range was chosen to be from ten cycles to one or two kilocycles per second.[1, 5] Over this wide frequency range, transformers have certain limitations that require the occasional use of

"isolators" to make them operate properly from zero frequency (d-c) up to the limit of one or two kilocycles per second (see Fig. 3).

The circuit impedance base must be kept low and the power level reasonably high to minimize stray coupling and stray pickup problems and enable the quick set-up of a wide variety of circuits. Unintentional impedances, such as the output impedance of a voltage amplifier, must be kept below a few ohms in series elements and above several hundred thousand ohms (or in some cases 10 or 20 megohms) in the case of shunt impedances. It is difficult to keep the ratio of the series-to-magnetizing impedance of a transformer sufficiently low over such a wide operating frequency range. This, together with the tendency of transformer coupled feedback circuits to "motor-boat," constitute the major difficulties encountered in the use of transformers. The "motor-boating" is only important when transient solutions are being obtained by repeating cyclically the forcing functions at the basic 10-cycle-per-second repetition rate in order to obtain a standing-wave solution on an oscilloscope. It can be overcome by applying the transient just once and recording the solution photographically or with a magnetic-tape recorder.

All of the electronic equipment is required to have less than one per cent amplitude distortion throughout the operating range. Furthermore, the frequency response of the equipment must not vary more than .1 decibel throughout the frequency range of the computer. The response is considered to be satisfactory if the phase shift is less than one degree throughout the entire frequency range. However, since these devices are frequently used in cascade and feedback circuits, the phase and amplitude characteristics must be carefully controlled to eliminate oscillations at both high and low frequencies. To prevent low frequency "motor-boating," all electronic equipment except the initial forcing-function generators was designed for operation from zero frequency (d-c) to the upper frequency limit of the computer. The high-frequency limit of all equipment was extended as far as practicable to make it easier to eliminate high-frequency oscillations.

The electronic elements discussed above have been described in detail.[7,17] It might be of interest, however, to discuss briefly the circuit elements for non-linear functions.

Generator of an Arbitrary Function of the Dependent Variable. One of the generators of non-linear functions of the dependent variable (Type 1) uses crystal diodes and batteries in the manner shown in Fig. 8. The adjustable resistors are relatively large compared to the forward impedance of the crystals, so that the variations of this impedance with current do not affect the over-all impedance appreciably. If the switches are all thrown to the left in Fig. 8, each parallel branch will not conduct current until the terminal voltage rises above the battery voltage of that branch. The resulting current-voltage characteristic is composed of a series of straight lines, and the slope of successive segments decreases as the voltage rises. Such a characteristic is shown in curves A of Fig. 8. The two curves labeled A are both of the same characteristic, but they indicate that both the positive and negative parts of a function may have entirely different characteristics if desired.

If the switches are thrown to the right, a different characteristic results. Each branch produces a circulating current which passes through R_0.

The battery voltage E_0 gives zero terminal voltage when no external current enters the terminals. For a given applied voltage, the current is determined by the effective resistance of all the branches in parallel. As soon as the terminal voltage exceeds the lowest branch battery voltage, the current in that branch ceases to flow, and the effective impedance of the device is raised. When the terminal voltage exceeds the highest battery voltage, the impedance becomes equal to R_0, the maximum value. The resulting current-voltage characteristic is one of constantly increasing slope, as shown in curve B. By combining branches of each type, a characteristic may be achieved whose slope is alternately increasing and decreasing, provided it always remains positive. Curve C, Fig. 8, is typical.

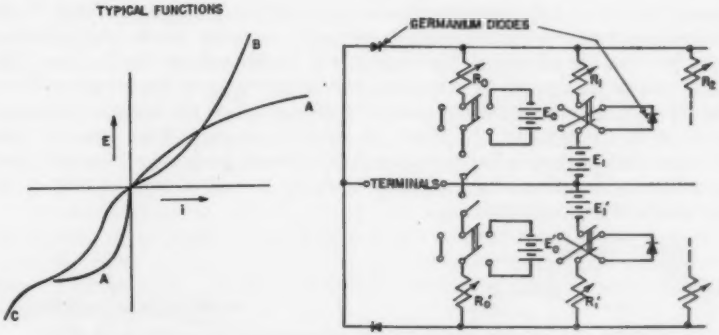


FIG. 8. Schematic Diagram—Dependent Variable Device Type 1.

It has been assumed that the voltage applied to the device is proportional to the dependent variable and that the current is proportional to an arbitrary function. If, however, the generator is driven by a current source, then the voltage across the device represents the arbitrary function. In this case, the type of characteristic will be reversed because of the interchange of axes (see Fig. 8).

The generator which is used with the computer has 22 parallel branches, 11 for positive voltages and 11 for negative voltages. A current-voltage characteristic composed of 22 straight-line segments can then be obtained. Such a characteristic can be used to approximate many typical functions with surprisingly good accuracy. Simpler and more specialized versions of the generator have also been developed in the form of abrupt current and voltage limiters.

The other arbitrary function generator (Type 2) is based upon a principle developed elsewhere.[19,20] It employs a cathode-ray tube and photoelectric cell. With no signal, the electron beam is directed to one side of the tube. The photo-cell output is amplified and applied to the plates with the proper polarity to drive the beam to the other side. If an opaque template is placed on the screen of the tube, as shown in Fig. 9, the beam spot will not appear until the light reaching the photo-tube produces a large enough signal to deflect the beam above the edge of the template.

The generator has several notable limitations. There is some distortion for which compensation is difficult, since the spot is not infinitely small and because the amplifier gain, the spot intensity, the height of the pattern, and the quiescent position all may vary from time to time. In addition, the halo on the screen contributes an appreciable signal, which may cause clipping of sharp, high peaks of the template, since the halo signal can be picked up on both sides of the peak with the spot completely hidden. Since the actual spot is on the rear of the cathode-ray screen while the observed pattern is on the front, there will be a third source of error in the resulting parallax. Careful proportioning of over-all height and width of the pattern, location of quiescent position, and adjustment of gain minimize these difficulties, but a definite accuracy limitation exists. The inability of the beam to rise or fall abruptly introduces another limitation. Thus, if the dependent variable passes rapidly through a region where the arbitrary function changes abruptly, the output will lag behind the ideal curve. This effect can be minimized by reducing the vertical scale of the template. Since the time lag is very small compared to the period of the highest frequency used on this computer, the effect on solutions is negligible. Despite these inherent limitations, it has been possible to develop a device that will form an arbitrary function of a dependent variable to within one per cent of the function's crest magnitude.

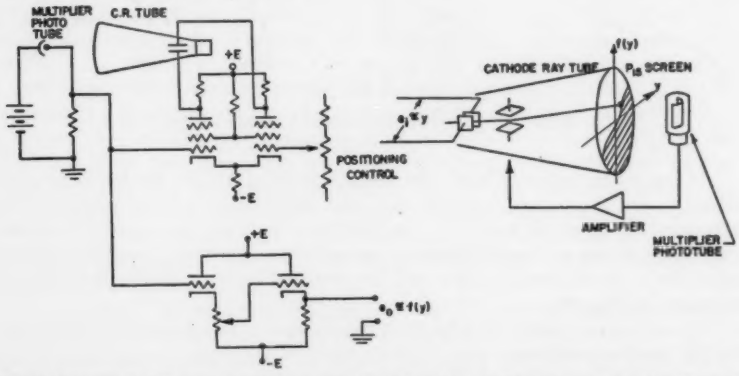


FIG. 9. Schematic Diagram of Arbitrary Forcing Function of Dependent Variable—Type 2.

Multiplier. The multiplication of two arbitrary voltages can be accomplished in many ways. The method which has been adopted for the CIT Computer uses double modulation followed by detection.[7] A balanced modulator produces sidebands by modulating a carrier with one of the product terms. The resulting voltage is of the form $E_1 \cos(\omega_c t)$. This voltage is used as a carrier for a second modulator which gives a sideband output, $E_1E_2 \cos(\omega_c t)$. The carrier voltage is, of course, suppressed through use of a balanced modulator in both cases. Addition of a carrier voltage permits detection, with the output voltage proportional to the product, E_1E_2. The d-c voltage resulting from detection of the added carrier is balanced by detection of another carrier signal and subtraction of the outputs. For con-

venience, a varistor bridge is used for the first modulator and vacuum tubes for the second modulator. Detection is accomplished with germanium diodes, thus minimizing the power supply requirements of the unit. Isolation between input and output is automatically supplied by the high-frequency transformers. One of the input voltages is not isolated from the system ground. If such isolation is required for both inputs, an additional isolator must be used. Care must be taken to minimize phase shift in the sidebands since this results in a phase shift of the detected signal. The first modulator, with untuned transformers, performs very well in this respect. Since tuned circuits are required in the second modulator for several reasons, the phase shift is minimized by over-coupling of the transformer to the plate circuit of the modulator. In the case of the multiplier, the accuracy is much better than the over-all one per cent requirement. The groups of multipliers and d-c isolators in the computer are supplied by a common high-frequency generator, thus reducing the space required for each unit.

Present Fields of Application. Although the computer has been in active service for almost two years and has been applied to a wide range of problems in many fields of engineering and science, it is not yet possible to predict its complete range of usefulness. It may be more appropriate to merely list some of the more important types of problems for which it has been used.

In the field of servomechanisms and automatic control,[9, 10, 12, 14, 17] the computer makes possible the rapid solution of control equations describing even the more comprehensive cases of missiles or autopilots with several degrees of freedom, and non-linearities in both the control system and the aerodynamic equations.

General analyses have been made of many simple and complex linear and non-linear ordinary differential equations pertaining to mechanical vibration analysis.[8, 11, 16] Equations have been solved occurring in such fields as packaging of delicate apparatus, the investigation of the design and performance of turbine generators, short-circuit torques, airplane landing shock, acceleration through resonance, and earthquake shock effects on buildings.

A practical method has now been developed for solving accurately and rapidly the more general partial differential equations of elastic beams and plates.[18] This facilitates the solution of the general types of transient beam and plate problems, as well as static loading and steady-state vibration problems.

The computer has been used for the solution of several linear thermal conductivity problems involving time and two space variables, and of non-linear conductivity problems involving time and one space variable. Complex boundary conditions and heat generation within the media can be readily handled.

Other fields for which the California Institute of Technology Electric Analog Computer has so far been used include compressible fluid flow studies, complex electric circuit analysis, aircraft landing stability analysis, basic mechanical system backlash and non-linear friction analysis, rideability analysis of automobiles and railroad cars, and many others.

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[21]

TABLE

Basic Elements of the California Institute of Technology
Electric Analogue Computer

A. *Ten Passive Circuit Elements Cabinets*

1. 100 high "Q" inductors	0 to 1.0 henrys in 0.001 henry steps
2. 20 high "Q" inductors	0 to 1.50 henrys in 0.01 henry steps
3. 100 resistors	0 to 7,000 ohms in 2 ohm steps
4. 20 resistors	0 to 500,000 ohms in 200 ohm steps
5. 60 capacitors	0 to 1.0 mfd in 0.01 mfd steps
6. 40 capacitors	0 to 4.0 mfd in 0.01 mfd steps
7. 10 capacitors	0 to 80 mfd in 10 mfd steps
8. 25 special transformers*	

B. *Five Plugboards*

1. Plugboards for all elements of two circuit elements cabinets
2. Receptacle patch-board for 500 element connections and 25 main busses

C. *Two Forcing Functions Cabinets*

1. 10 sinusoidal variable-frequency and phase-position-voltage sources (0 to 50 volts, one ampere, one ohm)

2. 12 separate square wave voltage sources (0 to 12 volts, or one 72 volt source in 2 volt steps)
3. 5 arbitrary functions of independent variable
4. 5 arbitrary current sources (controllable from any voltage source)

D. *Two Metering Desks*

1. 3 dynamometer-type meters and 2 vacuum tube meters for metering sinusoidal current, voltage, power, etc.
2. 3 cathode-ray oscillographs for transient solution (current, voltage, or charge)
3. 2 three-wire metering circuits (and selector system) for metering all passive circuit elements and source forcing functions
4. 2 manual metering selector circuits for 16 of 25 main busses
5. Circuits for connecting 36 synchronous switches to any of 26 main busses

E. *Amplifier Cabinets* (portable)

1. 5 negative gain (0 to 50) d-c amplifiers (15 ohm impedance), each having an RC time-delay circuit for servo-problems
2. 15 positive gain (0 to 100) d-c amplifiers with input and one ohm output impedance
3. 10 d-c isolators and preamplifiers

F. *Multiplier Cabinets* (portable)

1. 10 multipliers with isolated inputs*

G. *Arbitrary Functions of Dependent Variable* (portable)

1. 3 voltage limiters
2. 2 current limiters
3. 1 arbitrary function of dependent variable—Type I
   10 arbitrary functions of dependent variable—Type II

* Additional elements are contemplated. The Table lists only elements of the computer in its present operating form.

RECENT MATHEMATICAL TABLES

658[A–D, Q, R].—ITALY, ISTITUTO IDROGRAFICO DELLA R. MARINA, *Tavole Logaritmiche a Cinque Cifre Decimali.* Sixth ed. Genoa, 1941, xxii, 512. 19.7 × 27.8 cm. Full cloth, and gilt titles.

Contents: **T. I** (p. 1–57).—Mantissae of logarithms and cologarithms of numbers from 1 to 10 080; **T. II** (p. 60–179)—Logarithms of the absolute values of the trigonometric functions 0(1″)2°; **T. III** (p. 182–361)—Logarithms of the absolute values of the trigonometric functions 0(15″)90°; **T. IV** (p. 364–382)—Addition and subtraction logarithms; **T. V** (p. 384–428)—Values of the natural trigonometric functions 0(1′)90° or 0(4ˢ)6ʰ; T. **VI–XIII** (430–464)—Conversion of common logarithms to natural, and vice versa; n^2, n^3, $n^{\frac{1}{2}}$, $n^{\frac{1}{3}}$, ln n, 1000 n^{-1}, πn, $\frac{1}{4}\pi n^2$, n = 0(1)1020; table of arcs, altitude of sector, chord for 0(1°)180°, also arcs corresponding to 0(1′)60′, 0(1″)60″; conversion of centesimal grades, minutes and seconds into sexagesimals; conversion of arcs into time and vice versa; conversion of sexagesimal minutes and seconds into decimals of degrees, and vice versa; geodetic elements relative to the international ellipsoid; values of international normal gravity, g, for ϕ (geogr. latitude) = 0(15′)90°. Constants and units of measure (p. 467–480). Formulae (p. 483–512)—algebra, plane analytic geometry, differential and integral calculus, plane and spherical trigonometry, series, interpolation, astronomy, geodesy, least squares.

EDITORIAL NOTE—The first edition of this work was published in 1913; the second, 1916; third, 1922; fourth, 1930; fifth, 1937. There was a "nuova edizione, ridotta," 128 p., in 1939, and a copy may be seen in Library of Congress.

659[A–E, J–L].—JOHN BORTHWICK DALE, *Five-figure Tables of Mathematical Functions comprising Tables of Logarithms*, Powers of Numbers, *Trigonometric*, Elliptic, and other *Transcendental Functions*. Second edition, London, Edward Arnold & Co., 1949, viii, 121 p. 13 × 21.6 cm. 6 shillings. New York, Longmans, Green & Co., $1.50.

This popular work, first published in 1903 (xv, 92 p.) was reprinted in 1905, 1908, 1909, 1911, 1913, 1917, 1919, 1922, 1927, 1932, 1937, 1942, 1943, 1945. It was prepared with the idea of meeting the requirements of workers in Physical Science and Applied Mathematics. For the new edition the type has been reset, two new tables have been added (IA (p. 14–18), logarithms of numbers from 1000 to 2999, and XVIIA (p. 88–90) logarithms of hyperbolic functions), and other tables have been rearranged in place and in forms more suitable for ready use. It would seem as if many errors had been made in resetting the tables; here are five random illustrations: $\ln 4 = 1.38629$ (instead of 1.38269); $3.98^{\frac{1}{2}} = 1.5848$ (instead of 1.5484); $4.55^{\frac{1}{2}} = 1.6571$ (instead of 1.6751); $5.49^{\frac{1}{2}} = 1.7641$ (instead of 1.7461); p. 112, last col. $x = .87$, .83424 (instead of .83244).

The Tables are still numbered I–XXXIX: I–VII (p. 10–53), logs of numbers, antilogs, squares, square roots, cubes, cube roots, reciprocals; VIII–XIV (p. 54–81), the natural trigonometric functions and their logs, circular or radian measure; XV–XVIII (p. 82–91), natural logs—to base e, nat. log $10^{\pm n}$, exponential and hyperbolic functions, log e^n; XIX (p. 92), $gd^{-1}\theta$; page 93 is blank; XX–XXII (p. 94–102), $F(\theta, \phi)$, $E(\theta, \phi)$, complete elliptic functions (the rounding error of $F_1(88°)$ in the final digit, 1 instead of 2, *MTAC*, v. 3, p. 262, still persists); XXIII–XXV (p. 103–105), gamma function, logs of factorials, zonal surface harmonics, $P_n(x)$, $n = 1(1)7$, $x = 0(.01)1$; XXVI–XXVII (p. 106–107), $J_0(x)$, $J_1(x)$, $x = 0(.1)15$, $J_n(x)$, $n = 0(1)12$, $x = 0(1)20$ (the error in $J_0(5.9)$ in the first ed. has been corrected); XXVIII (p. 108–109), $I_n(x)$, $n = 0(1)11$, $x = 0(.2)5$, Lodge, 1889; XXIX (p. 110), $J_n = 0$, first 9 roots $n = 0(1)5$ (here are 27 end-figure errors of 1 to 25 units); first 6 roots of $J_0(x)$ and $J_1(x)$, $j_{0,s}/\pi$ and $j_{1,s}/\pi$ (Stokes 1851 tables); XXX (p. 110), $J_n(x)$, $n = \nu + \frac{1}{2}$, $\pm 2n = 1(2)11$; XXXI (p. 111), Erf(x); XXXII (p. 112–113), Ei$(\pm x)$, Ci(x), Si(x); XXXIII–XXXIV (p. 114–116), binomial coefficients for interpolation by differences, and proportional parts; XXXV–XXXVII (p. 117), reciprocals of factorials, Bernoulli numbers $(B_1–B_{20})$; conversion of time and angular measure; XXXVIII (p. 118), decimal equivalents, fractions of Julian year of $365\frac{1}{4}$ days; XXXIX (p. 119–121), numbers used in calculations, Euler's numbers, Euler's constant; 10D values, for $n = 1(1)20$, of $S_n = 1^{-n} + 2^{-n} + 3^{-n} \cdots$, $s_n = 1^{-n} - 2^{-n} + 3^{-n} - \cdots$, $T_n = 1^{-n} + 3^{-n} + 5^{-n} + \cdots$; English and metric equivalents.

Dale does not give the source of any of his tables.

In conclusion it may be noted that in the first ed. [1913, "sixth impression"], p. 27, arg. 993, *for* 3.9766, *read* 9.9766; p. 91, l. 20, *for* 3.0205, *read* 3.0255.

R. C. A.

660[B–E, M, N].—ANDRÉ SAINTE-LAGÜE, *Mathématiques*, p. A30–A110, in *Techniques de l'Ingénieur, directeur général de la rédaction* C. MONTEIL. I: *Généralités*. Paris, Techniques de l'Ingénieur, 1947, 24.3 × 29.4 cm.

There are various tables of powers, quarter squares, interest, logs, antilogs, natural and hyperbolic trigonometric functions, etc., p. A30–A60, 43 p.; table of indefinite integrals, p. A80–5 to A80–14. Nomography, p. A110–12 to A110–18.

661[C, D, K].—ANDRÉ DANJON, *Tables des Fonctions Trigonométriques. Valeurs naturelles à 6 décimales de centième en centième du degré nonagésimal*. Paris, Librairie Hachette, 1948, ii, 126 p. 18.4 × 27.2 cm. Stiff cover. 850 francs.

This is the third publication, within a year, in which are tables of natural trigonometric functions in the nonagesimal system with the arguments taken at interval 0°.01. COMRIE's

tables (RMT 631), and NBSCL (RMT 662) are the others. The further publication of tables using minutes and seconds, instead of decimals of a degree, is likely to become increasingly rare. Thus we are now harking back to tables such as nearly 350 years ago HENRY BRIGGS thought were the most useful he could prepare.

Here are 6D tables (p. 6–95), with Δ, of sin, csc, tan, cot, sec, and cos. In connection with sines and cosines linear interpolation can be used throughout, and in the case of tangents and secants from 0 to 75°.8; the method for dealing with later angles is indicated. If one is content with 6S, linear interpolation is legitimate up to 87°. On p. 5 are interpolation tables for trigonometric functions, including S and T. There is also a loose card for 11(1)200(100)900 in a proportional parts table.

The present tables were prepared from a 10D manuscript. The author states that the proofs were carefully compared with the tables in *Trigonometria Britannica*, 1633, of Briggs and for each 0°.05 with ANDOYER, *Nouvelles Tables Fondamentales* (1915–1918).

On p. 96–99 are tables for the conversion of (a) sexagesimal fractions of a degree into decimal fractions; (b) decimal fractions of a degree into sexagesimal fractions; (c) hours into degrees and decimal fractions of a degree; (d) degrees into hours. A 6D table of logs 1000(1)9999 occupies p. 100–117, and various mathematical formulae and constants, p. 119–125. On p. 126 are reprinted two statistical tables, Normal distribution of Laplace-Gauss, and Distribution of χ^2 from R. A. FISHER & F. YATES, *Statistical Tables*, third ed., 1948.

The work presents a neat appearance. The one-column "Directions for the use of the tables" is in four languages: French, Spanish, English and German.

R. C. A.

662[D].—NBSCL, *Table of Sines and Cosines to Fifteen Decimal Places at Hundredths of a Degree.* (*NBS Applied Math. Series*, no. 5.) Washington, Govt. Printing Office, 1949. viii, 95 p. For sale by the Superintendent of Documents, Washington, D. C. 40 cents. 19.8 × 25.9 cm.

We have here an original valuable table of sines and cosines to 15D, δ^2, throughout the quadrant, at interval 0°.01. A preliminary manuscript to 18D was checked by differencing and then rounded to 15D and typewritten. The final manuscript was again checked by differencing.

The introduction by H. E. SALZER includes a discussion of direct and inverse interpolation, with examples, and the usual interpolation tables, of $E(p)$ and $F(p)$, are given at the end of the volume, p. 94–95.

The appearance of the volume would have been decidedly improved by the omission of the tautological headings, "Table of Sines and Cosines," on 90 pages.

It is more than 300 years ago since tables such as these were first computed and published. About 1601 HENRY BRIGGS (1561–1631) finished the calculation of a remarkable series of tables at interval 0°.01, throughout the quadrant, including those of sines and cosines to 15D, Δ. These tables were posthumously published by HENRY GELLIBRAND (1633). We have already indicated (*MTAC*, v. 1, p. 129–130) that Briggs was led to the decimal choice of such parameter by perusal of a publication of Vieta. In the first part of this 1633 volume (p. 43–44) there is also a table of sines for each 0°.625 in the quadrant to 19D, Δ.

Many years earlier than Briggs RHETICUS (died 1576) computed an extraordinary table of sines for each 10″ of the quadrant, 15D, Δ^2 *or* Δ^3; compare N109. Hence we may say that this table includes the values of sines and cosines to each 0°.125 of the quadrant. This table was first published by PITISCUS[2] in 1613 along with a Rheticus table of sines at interval 1″, from 0 to 1° and from 89° to 90°, 15D, Δ^3, and hence giving values at interval 0°.0625 in this range.

In the work under review there is no reference to these Rheticus tables or to the ANDOYER[3] tables (1915) of sines and cosines covering the same range, 15D, Δ, and hence including values to each 0°.125 of the quadrant.

There are three further tables of sines and cosines to at least 15D, as follows: by HERRMANN[4] (1848), [1°(1°)89°; 30D], and reprinted, p. 92–93 in the work under review; by J. T.

PETERS[5] (1911), [0(10′)45°; 21D], and therefore including a table at interval 0°.5; and SPENCELEY & SPENCELEY[6] (1947), [1°(1°)90°; 25D].

On checking the first 50 values each of sines and cosines of the NBSCL table and that of Briggs, it was found that among the 50 sine values of the latter 42 differed from the former only in the fifteenth place, and by 1 to 4 units (20 unit errors, 5 2-unit errors, 5 3-unit errors, and 12 4-unit errors). Among the values of the 50 cosines there were only 11 unit errors in the fifteenth decimal place.

R. C. A.

[1] H. BRIGGS & H. GELLIBRAND, *Trigonometria Britannica*, Gouda, 1633.
[2] G. J. RHETICUS, ed. by B. PITISCUS, *Thesaurus Mathematicus*, Frankfort, 1613.
[3] H. ANDOYER, *Nouvelles Tables Trigonométriques Fondamentales*, v. 1, Paris, 1915.
[4] HERRMANN, "Bestimmung der trigonometrischen Functionen aus den Winkeln und der Winkel aus den Functionen, bis zu einer beliebigen Grenze der Genauigkeit," Akad. d. Wissen., Vienna, *Math. naturw. Kl., Sitzb.*, v. 1, "second unchanged ed.," 1848, p. 477–478; a similar table of tangents and cotangents is given on p. 479–480. In this v. the pages are numbered consecutively from the beginning to the end. These page numbers agree with those given in the *Royal Soc. Cat. Sci. Papers*. Yet the references given by Mr. SALZER, p. v, are entirely different, namely: pt. IV, p. 176–177, and 178–179. It would therefore seem as if the five parts of the volume were originally issued with separate paging but when assembled in a volume were paged consecutively.
[5] J. T. PETERS, "Einundzwanzigstellige Werte der Funktionen Sinus und Cosinus zur genauen Berechnung von zwanzigstelligen Werten sämtlicher trigonometrischen Funktionen eines beliebigen Arguments sowie ihrer Logarithmen," Akad. d. Wissen., Berlin, *Phys.-math. Cl., Abh.*, 1911, p. 12–18.
[6] G. W. & R. M. SPENCELEY, *Smithsonian Elliptic Functions Tables*. Washington, 1947.

663[D, H, M].—HARRY D. HUSKEY, "On the precision of a certain procedure of numerical integration"; Appendix: D. R. HARTREE, "Note on systematic rounding-off errors in numerical integration," NBS, *Jn. of Research*, v. 42, 1949, p. 57–62. 19.7 × 26 cm.

Summary: An example of numerical integration (illustrated by three trigonometric tables) is given and shows very systematic effects in the less significant digits. This lack of randomness gives rounding-off errors that exceed the predicted standard deviation by a factor of three.

The example considered shows that systematic rounding-off errors can occur in numerical integration, irrespective of the number of digits kept in the contributions to the integral. In the Appendix this phenomenon is examined, and criteria are set up to detect the cases in which it may arise to a serious extent.

664[D, I].—JØRGEN RYBNER & K. STEENBERG SØRENSEN, *Tabel til Brug ved Addition af Komplekse Tal. Table for use in the Addition of Complex Numbers*. Copenhagen, Jul. Gjellerups Forlag, 1948, xiv, 95 p. 23.6×31.5 cm. 20 Danish crowns. American agent: Scandinavian Book Service, P.O. Box 99, Audobon Sta., New York 32. $5.50.

The addition of two complex numbers can be carried out by means of a table by dividing by the number having the larger numerical value (in KENNELLY'S notation $\underline{/\phi} = e^{i\phi} = \cos\phi + i\sin\phi$):

$$z_3 = z_1 + z_2 = r_1\underline{/\phi_1} + r_2\underline{/\phi_2} = r_1\underline{/\phi_1}[1 + (r_2/r_1)\underline{/\phi_2 - \phi_1}],$$
$$r_2 \leq r_1 = r_1\underline{/\phi}(1 + r\underline{/\phi}) = r_1\underline{/\phi_1}\cdot R\underline{/\alpha} = r_1 R\underline{/\phi_1 + \alpha}.$$

Subtraction is reduced to addition by changing the negative sign of z_2 into an alteration of its phase angle by $\pm 180°$:

$$z_3 = r_1\underline{/\phi_1} - r_2\underline{/\phi_2}, \qquad = r_1\underline{/\phi_1} + r_2\underline{/\phi_2 \pm 180°}.$$

The table represents the function

$$R/\underline{\alpha} = 1 + r/\underline{\phi}$$

giving R(5D) and α(3D) as functions of r and ϕ, for $r = 0(.01)1$, $\phi = 0(1°)180°$.

$R^2 = 1 + r^2 + 2r\cos\phi$, $\quad \sin\alpha = r\sin\phi/R$, $\quad \tan(\tfrac{1}{2}\phi - \alpha) = [(1-r)/(1+r)]\tan\tfrac{1}{2}\phi$.

The computations were started by calculating R and α directly from these formulae at intervals .05 for r, and 5° for ϕ. These calculations were carried out by means of the seven-place trigonometrical tables of PETERS,[1] R being determined to 7D and α to 5D, that is, in both cases two more places than given in the final table. The other values of the table were found by interpolation.

Pages 2–93 are devoted to tabulation of R and α for $r = 0(.01)1$, $\phi = 0(1°)180°$. Differences d_r, d_ϕ are given throughout. At the bottoms of the R and α pages are the interpolation formulae:

$$R(r_0 + xh,\quad \phi_0 + yk) = R_{00} + xd_r + yd_\phi + xyd_{r\phi}{}^2 + B''(x)d_{rr}{}^2 + B''(y)d_{\phi\phi}{}^2$$
$$\alpha(r_0 + xh,\quad \phi_0 + yk) = \alpha_{00} + xd_r + yd_\phi + xyd_{r\phi}{}^2 + B''(x)d_{rr}{}^2 + B''(y)d_{\phi\phi}{}^2$$

and on each page are the ranges of values of $d_{r\phi}{}^2$, $d_{rr}{}^2$, $d_{\phi\phi}{}^2$. On p. 95 is a table of interpolation coefficients $B''(u) = -\frac{1}{2}u(1-u)$, $u = x$ or y.

The introductory pages include a number of worked out examples illustrating the table's use; for example: the values of R and α are found when $r = .5082$, $\phi = 136°.59$.

It is obvious that great care was taken to attain accuracy in preparation of the volume.

In an earlier work by Professor Rybner,[2] which we reviewed, *MTAC*, v. 3, p. 174–175, there was a nomogram for $R/\underline{\alpha} = 1 + r/\underline{\phi}$ but the present table was necessary because it was not found possible to construct the corresponding nomograms, with an accuracy sufficient for practical calculations.

A small table of $R/\underline{\alpha} = 1 + r/\underline{\phi}$ was given by F. EMDE[3] but the angles are measured in fractions of a right angle, 0(.05)2, which is inconvenient, since this unit is not commonly used.

R. C. A.

[1] J. T. PETERS, *Siebenstellige Werte der trigonometrischen Funktionen von Tausendstel zu Tausendstel des Grades*. Berlin, 1938.

[2] J. RYBNER, *Nomograms of Complex Hyperbolic Functions*. Copenhagen, 1947.

[3] F. EMDE, *Tables of Elementary Functions. Tafeln elementarer Funktionen*. Leipzig, 1940. American reprint 1945, p. 32–33. See *MTAC*, v. 1, p. 384–385. The table is also given in JAHNKE & EMDE, *Tables of Functions*, e.g. 1945, p. 18–19 of the second part.

665[E].—HARVARD UNIVERSITY, COMPUTATION LABORATORY, *Annals*, v. 20, *Tables of Inverse Hyperbolic Functions*, By the staff of the Laboratory, H. H. Aiken, director. Cambridge, Mass., Harvard University Press, 1949, xx, 290 p. 19.5 × 26.7 cm. $10.00. Offset print.

These tables give 9D values with first and second forward differences of the inverse hyperbolic sine, cosine, and tangent. More precisely, **T. I** (p. 3–47), from which **T. II** and **T. III** were derived, gives $\tanh^{-1}x$ for $0 \leq x < 1$. For $x < .9$ the values of the argument are

$$x = 0(.001).5(.0005).75(.0002).9.$$

Only 39 percent of the table is devoted to 90 percent of the unit interval. The singularity of $\tanh^{-1}x$ at $x = 1$ forced the tabulation for $.9 < x < 1$ to proceed by finer and finer intervals in order to preserve the adequacy of second order interpolation. The final entry is for $x = .99999$.

T. II (p. 51–66) is straightforward and gives $\sinh^{-1}x$ for

$$x = 0(.002)3(.005)3.495.$$

T. III (p. 69–103) gives $\cosh^{-1}x$ for $1 < x < 3.5$. Here trouble is encountered at $x = 1$ so that one-half of the table has to be devoted to 6 percent of the entire range of x. Beyond

$x = 1.15$ the argument values are

$$x = 1.15(.0005)1.4(.001)1.8(.002)3.498.$$

Finally **T. IV** (p. 107–290) gives $\sinh^{-1} x$ and $\cosh^{-1} x$ for $3.5 \leq x \leq 22980$ at various intervals ranging from .005 to 20 keeping the second differences under about 2000 units in the 9th decimal place.

The accuracy of the whole volume is to within .9 units in the 9th decimal. The computation time was only 12 days, of 24 hours each. This represents an increase in speed of the Automatic Sequence Controlled Calculator due to an improved printing mechanism.

The fact that the table was produced by automatic methods accounts for a number of its features which would have puzzled table makers of a decade ago. For example **T. IV** has the luxurious feature of tabulating two functions so nearly alike that their values differ by only a few hundred units in the last place of decimals and their second differences are practically indistinguishable. This goes on for more than 150 pages and would be an unthinkable waste of time and paper, not to mention an added source of typographical error, had the table been hand-made. In fact, by past standards an extensive table of these three functions has no doubt been considered unworthy of much effort in view of the fact that their values can be obtained from a good table of logarithms.[1] At any rate the bibliography of 17 titles[2] (p. xvi–xx) included mostly trivial tables. The well known table of Hayashi[3] is perhaps the only one approaching the present one in extent. The former table, however, is poorly arranged for these functions and is unreliable. The present volume, therefore, represents a significant advance in the tabulation of hyperbolic functions.

D. H. L.

[1] $\operatorname{Sinh}^{-1} x = \ln[x + (x^2 + 1)^{\frac{1}{2}}] = \operatorname{csch}^{-1}(1/x)$, $\quad -\infty < x < \infty$
$\cosh^{-1} x = \pm \ln[x + (x^2 - 1)^{\frac{1}{2}}] = \operatorname{sech}^{-1}(1/x)$, $\quad 1 < x < \infty$
$\tanh^{-1} x = \frac{1}{2} \ln[(1 + x)/(1 - x)] = \coth^{-1}(1/x)$, $\quad -1 < x < +1$.

[2] After listing the tables of inverse hyperbolic functions in M. BOLL, *Tables Numériques Universelles*. Paris, 1947, p. 475, there is the remark: "This single page of tables is replete with errors." See *MTAC*, v. 2, p. 336–338; v. 3, p. 466–467.

[3] K. HAYASHI, *Sieben- und mehrstellige Tafeln der Kreis- und Hyperbelfunktionen*, Berlin, 1926, p. 9–201.

666[F].—D. H. LEHMER, "On the converse of Fermat's theorem," *Amer. Math. Mo.*, v. 43, 1936, p. 347–354; v. 56, 1949, p. 300–309.

Fermat's theorem states that if a is any integer, and p a prime, then $a^p - a$ is divisible by p. For $a = 2$, the strict converse, attributed to the ancient Chinese, would assert that if n divides $2^n - 2$, then n is a prime. SARRUS in 1819 disproved this by the counter-example $n = 341 = 11 \times 31$; $2^{341} - 2$ is divisible by 341. A true converse of Fermat's theorem, due to LUCAS, states that if n divides $a^x - 1$ for $x = n - 1$, but not for x a proper divisor of $n - 1$, then n is prime. In both theoretical and practical work it is frequently required to settle whether a fairly large number is prime or not. No general usable test is available (WILSON's theorem is unusable). "One may ask for a criterion which is almost characteristic of primes and then tabulate those relatively few composite numbers which also satisfy this condition." The false Chinese converse is such a criterion. But to find the composite exceptions, like 341, factorizations of large numbers are frequently demanded. The author's long experience in this enabled him in the first paper to publish a list of composite exceptions[1] between 10^7 and 10^8 whose least factor exceeds 313. The smallest prime factor of each is listed. In the second paper the list is extended[2] from 10^8 to $2 \cdot 10^8$. The Army Ordnance's ENIAC all-electronic digital computer made the work possible in a remarkably short time. Incidentally it is proved that there is an unlimited number of n's dividing $2^n - 2$ that are the products of three primes.

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[1] This list contained 526 entries believed to contain all exceptional numbers; no number having more than two prime factors, each in excess of 313. In the second paper it was dis-

covered after recomputation that in this list there were 2 entries to delete and 7 to insert (*MTAC* v. 2, p. 279) so that there are exactly 531 exceptional numbers in question less than 10^8.—Editor.

[2] In the second list are 327 entries—320 two prime-factor numbers and 7 three prime-factor numbers. Thus there are in all 858 exceptional numbers $< 2 \cdot 10^8$.—Editor.

667[F].—D. H. Lehmer, "On the partition of numbers into squares," *Amer. Math. Mo.*, v. 55, 1948, p. 476–481.

$P_k(n)$ denotes the number of partitions of n into k integral squares $\geqq 0$. Partitions differing only by permutations of the squares are not counted as distinct. The author is concerned with the solutions of $P_k(n) = 1$. It has long been known that $P_k(n) > 0$ for $k \geqq 4$. The author solves completely $P_4(n) = 1$, and finds all numbers that can be partitioned into 4 squares in precisely 2 ways. For $P_5(n) = 1$ he finds the solutions $4^a m$, $a \geqq 0$, for $m \leqq 427$. Any further values of m exceed 10,000.

E. T. Bell

668[F].—Nils Pipping, "Diagonalkettenbrüche," Aabo, Finland, Akademi, *Acta, Mathem. et Phys.*, v. 16, no. 5, 1949, 23 p.

The conclusion of this paper (p. 16–23) presents a table of the expansion of $D^{\frac{1}{2}}$ in a diagonal continued fraction for non-square integers D from 501 to 1000. This is an extension of a previous table (*MTAC*, v. 3, p. 96). In 97 of the 491 expansions the diagonal continued fraction is regular.

D. H. L.

669[F].—D. Yarden & A. Katz, "Luaḥ sedarot nesiga linariyot binariyot misseder 3" [Table of binary linear recurring sequences of order 3]. *Riveon Lematematika*, v. 2, p. 54–55, 1948.

Four recurring sequences defined by

$$\begin{array}{llll} U_n = U_{n-2} + U_{n-3} & U_0 = 0, & U_1 = 0, & U_2 = 1 \\ V_n = V_{n-2} + V_{n-3} & V_0 = 3, & V_1 = 0, & V_2 = 2 \\ \bar{U}_n = -\bar{U}_{n-2} + \bar{U}_{n-3} & \bar{U}_0 = 0, & \bar{U}_1 = 0, & \bar{U}_2 = 1 \\ \bar{V}_n = -\bar{V}_{n-2} + \bar{V}_{n-3} & \bar{V}_0 = 3, & \bar{V}_1 = 0, & \bar{V}_2 = -2 \end{array}$$

are tabulated for $\pm n = 0(1)64$. The factorizations of the numbers are also given with the exception of $V_{62} = 2 \cdot 9803919989$. Prime values of these four functions are underlined.

D. H. L.

670[I].—H. E. Salzer, *Table of Coefficients for Inverse Interpolation with Central Differences*. NBS, *Mathematical Table*, no. MT 27. Second ed., June 1, 1949. Washington, D. C., ii, 15 p. 14.9 × 23.2 cm.

This table, previously reviewed in *MTAC*, v. 1, p. 315–316, is here reprinted with permission of the editors of *Jn. Math. Phys.* No changes were made in the text or tabular material.

671[K].—Harold Jeffreys, *Theory of Probability*. Second ed., Oxford, Clarendon Press, 1948, viii, 412 p. 15.4 × 23.5 cm. 30 shillings; New York branch office price $9.00. Appendix, p. 396–404, "Tables of K."

The author uses K as a generic term for the ratio of probabilities $P(q|\theta H)/P(q'|\theta H)$, where q is the initial hypothesis, q' the alternative, H the previous information, and θ the observational evidence. The actual seven tables given are double-entry tables, entered with values of n, or ν, and of K, to read values of χ^2, t^2 or z, depending upon the table. The par-

ticular function K is separately defined in each table. Each resulting table gives, of course, selected values of a specified mathematical function of two arguments, and does not depend for its calculation upon any particular theory of probability. The values of K are 1, $10^{-\frac{1}{2}}$, 10^{-1}, $10^{-\frac{3}{2}}$, 10^{-2}, in each table. In T. I–IV, n or $\nu(=n-1)$ ranges from 5 to 100,000 (save in T. II, where n starts at 7). The entries proceed by one unit up to 20, by ten units to 100, thence the numbers go by successive factors of 2 and 5, up to 100,000. The functional values to be read are to one place of decimals, and hence to two or three significant figures. In T. IIIA $\nu = 1(1)9$, T. IVA, $\nu = 1(1)8$, and in T. V, $\nu = 1(1)10(2)20$, 50, while z is read to 2D. In T. IIIA, IVA, and V, the values of K for $t = 0$, or $z = 0$, are also listed for the given values of ν. The argument, K, is related to n or ν and the function to be read, as follows:

T. I: $K = (2n/\pi)^{\frac{1}{2}} \exp(-\frac{1}{2}\chi^2)$; reading χ^2.
T. II: $K = \frac{1}{2}\pi n^{\frac{1}{2}}\chi \exp(-\frac{1}{2}\chi^2)$; reading χ^2.
T. III: $K = (\pi\nu/z)^{\frac{1}{2}}[1 + (t^2/\nu)]^{\frac{1}{2}(1-\nu)}$; reading t^2.

T. IIIA: $K^{-1} = (2/\pi)\int_0^\infty {}_1F_1\left\{\frac{1}{2}(1-n),\ \frac{1}{2},\ -\frac{n\nu^2x^2}{2(x^2+s'^2)}\right\}\exp\left\{-\frac{ns'^2v^2}{2(x^2+s'^2)}\right\}\frac{dv}{1+v^2}$

and ${}_1F_1(\alpha, \gamma, x)$, is the confluent hypergeometric function, $1 + \frac{\alpha x}{\gamma} + \frac{\alpha(\alpha+1)x^2}{2!\gamma(\gamma+1)} + \ldots$, $t = \nu^{\frac{1}{2}}x/s'$; reading t^2.
T. IV: $K = \frac{1}{2}\nu^{\frac{1}{2}}\pi t[1 + (t^2/\nu)]^{-\frac{1}{2}\nu}$; reading t^2.

T. IVA: $K^{-1} = 2\pi^{-1}\int_0^\infty {}_1F_1\left\{1 - \frac{1}{2}n,\ 1,\ -\frac{n(a^2+b^2)v^2}{2(2s'^2+a^2+b^2)}\right\}\exp\left\{-\frac{ns'^2v^2}{2s'^2+a^2+b^2}\right\}$

$\times \frac{dv}{1+v^2}$, where ${}_1F_1(\alpha, \gamma, x)$ is as given above, and $a^2 + b^2$ replaces x^2; reading t^2.

T. V: $K^{-1} = 2^{\frac{1}{2}}\pi^{-1}\int_0^\infty [(u^2+1)/(u^4+1)]u^n \exp\{\frac{1}{2}nb^2(1-u^2)\}du$ where $b = e^z$; reading z.

In general these tables serve to indicate with what degree of accuracy it is worth while to carry out the reduction of a given set of observations. Though differing in principle from the use of probability integrals, they lead to substantially the same numerical estimates.

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672[K].—N. V. SMIRNOV, "Table for estimating the goodness of fit of empirical distributions," *Annals Math. Statistics*, v. 19, 1948, p. 279–281. 17.4 × 25.2 cm. Reprinted from N. V. SMIRNOV, "On the estimation of the discrepancy between empirical curves of distribution for two independent samples," Moscow, Univ., *Bull. Math., série internationale*, v. 2, fasc. 2, 1939, p. 15–16. 14.7 × 21.8 cm.

This is a table of $L(z) = 1 - 2\sum_{\nu=1}^{\infty}(-1)^{\nu-1}e^{-2\nu^2z^2} = \sum_{-\infty}^{\infty}(-1)^\nu e^{-2\nu^2z^2} = (2\pi)^{\frac{1}{2}}z^{-1}$

$\times \sum_{\nu=1}^{\infty} e^{-(2\nu-1)^2\pi^2/8z^2}$ mostly 6D, for $z = .28(.01)2.5(.05)3$. The values vary from .000001 for for $z = .28$, to .999 99997 for $z = 3$.

It may be noted that in the *Annals*, p. 178 and 279 (W. FELLER) the expression for $L(z)$ in the first of the three forms given above is incorrect, since it has $e^{-\nu^2z^2}$. This error is copied in *Math. Reviews*, v. 9, p. 599. The second and third forms, given by Smirnov, are correct.

R. C. A.

673[K, L].—J. I. MARCUM, *Tables of Hermite Polynomials and the Derivatives of the Error Function.* U. S. Air Force. Project RAND no. P-90. Santa Monica, Cal., 29 December 1948, 241 leaves, text on one side only. Hektographed. 20.5 × 28 cm. Not available for distribution at present.

Introductory text: The error function may be defined by $\phi(x) = E'(x) = (2\pi)^{-\frac{1}{2}}e^{-\frac{1}{2}x^2}$. The n^{th} derivative of this function is then

$$\phi^n(x) = D^nE'(x) = (2\pi)^{-\frac{1}{2}}d^ne^{-\frac{1}{2}x^2}/dx^n.$$

The following tables give values of the derivatives of the error function so defined for

$$n = 1(1)10, \qquad x = 0(.01)12; 6S.$$

In the process of computing these tables, the Hermite polynomials, defined by

$$h_n(x) = (-1)^ne^{\frac{1}{2}x^2}d^ne^{-\frac{1}{2}x^2}/dx^n,$$

were generated as a by-product. The tables also include the values of these polynomials.

The specific purpose for computing these tables was for use in GRAM-CHARLIER series approximations to certain distribution functions for a sine wave plus random noise [see J. I. MARCUM, *A Statistical Theory of Target Detection by Pulsed Radar—mathematical appendix.* Project RAND, Report no. R-113, July 1, 1948 (restricted)]. For use in the Gram-Charlier series, n should go at least to 9 and x to at least 10 in interval .01. No combination of available tables could fulfill these requirements. Hence these tables were computed on the IBM machinery at Project RAND.

A table of exact values of $h_n(x)$ for $n = 2(1)6$, $x = 0(.01)4$ was given by N. R. JØRGENSEN *Undersøgelser over Frekvensflader og Korrelation.* Diss. Copenhagen, 1916, p. 196–205. There is also a table of $\phi^n(x)$ in T. C. FRY, *Probability and iis Engineering Uses.* New York, 1928, p. 456–457, $n = 1(1)6$, $x = [0(.1)4; 5D]$; and in New York, W.P.A., ms., $n = 1(1)14$, $x = [0(.1)8.4; 20D]$.

EDITORIAL NOTE: In the Marcum tables 5D uniformity in display for both $h_n(x)$ and $\phi^n(x)$ is attained by supplying extra columns for each function indicating the number of places the decimal point is to be moved to the right or to the left. Thus for $x = 12$ and $n = 10$ the first 6 figures in a 11 figure value for $h_n(x)$ are given, while for $\phi^n(x)$ we have a value to 27D.

674[L].—FREDERICK W. GROVER, *Inductance Calculations. Working Formulas and Tables,* New York, Van Nostrand, 1946, xiv, 286 p. $5.75. 15 × 23 cm.

Among 55 tables in this volume Dr. FLETCHER has already referred to T. 13, 36, 37, 40, 49–51 in his "Guide to tables of elliptic functions," *MTAC,* v. 3, p. 235, 238, 239, 253.

Auxiliary tables 3–4, p. 238–247 are of zonal harmonic functions and differential coefficients of zonal harmonics. T. 3: $P_n(x)$, $n = 2(1)8$, and $x^{-1}P_n(x)$, $n = 3(2)7$; T. 4: $P_n'(x)$, $n = 3(2)7$, Δ, Δ^2, or Δ^3; each table giving exact values for $x = 0(.01)1$. Such tables are also to be found in A. H. H. TALLQVIST, *Grunderna af Teorin för sferiska Funktioner.* Helsingfors, 1905.

R. C. A.

675[L].—D. R. HARTREE & S. JOHNSTON, "On a function associated with the logarithmic derivative of the gamma function," *Quart. Jn. Mech. Appl. Math.,* v. 1, 1948, p. 29–34. 15.3 × 23.4 cm.

Here is a table of $\mathrm{Re}[\psi(k) - \ln k + \frac{1}{2}k^{-1}] = \mathrm{Re}\psi(10 + i|k|) - \log|k| - |\gamma| \times \left[\frac{1}{1^2|\gamma| + 1} + \frac{2}{2^2|\gamma| + 1} + \cdots + \frac{9}{9^2|\gamma| + 1}\right]$, where $\psi(z) = \Gamma'(z)/\Gamma(z)$, for $\gamma = k^{-2} = [-1(.01) + 1; 8D]$, Δ^2. The values were evaluated to 10D. The maximum error in a tabulated value should not be more than .6 in the eighth decimal. Linear interpolation is adequate to give 6D accuracy.

Extracts from text

676[L].—VERA HUCKEL, *Tables of Hypergeometric Functions for Use in Compressible-flow Theory*, U. S. National Advisory Committee for Aeronautics, *Technical Note*, no. 1716, Oct. 1948, 13 p. 19.9 × 26.3 cm.

In the hodograph method of treating plane potential compressible flows the differential equation, originally obtained by CHAPLYGIN in his study on gas jets, plays a significant role. This paper tabulates various hypergeometic functions which arise as particular solutions of Chaplygin's differential equation. The tables should prove useful in the tabulation of other auxiliary functions which may arise in various compressible-flow problems. The adiabatic index for air has been taken as 1.4.

Chaplygin's differential equation is

$$(1) \qquad \tau(1-\tau)d^2Y_k/d\tau^2 + [(k+1) - (k+1-\beta)\tau]dY_k/d\tau + \tfrac{1}{2}\beta k(k+1)Y_k = 0$$

where $\beta = 2.5$, $M^2 = 5\tau(1-\tau)^{-1}$, M being the Mach number, and $Y_k(\tau) \to 1$, as $\tau \to 0$.

The tables of numerical values have been prepared for a selected number of the complete set of solutions of (1). These solutions extend the results of Chaplygin into the supersonic range and to negative values of the index k.

Tables 1–2 are for the functions Y_k, $k = -15(.5) + 15$, $\tau = [0(.01).5;\ 5D]$, with corresponding Mach numbers.

Tables 3–4 are for the functions $dY_k/d\tau$, $k = -15(.5) + 15$, $\tau = [0(.01).5;\ 4D]$, with corresponding Mach numbers.

REFERENCES

I. E. GARRICK & CARL KAPLAN, *On the Flow of a Compressible Fluid by the Hodograph Method II—Fundamental Set of Particular Flow Solutions of the Chaplygin Differential Equation*. NACA, *Report*, no. 790, 1944, 21 p.

S. A. CHAPLYGIN, *Gas Jets*, NACA, *Technical Memorandum*, no. 1063, 1944, 112 p. +3 leaves. Translated by S. REISS from the Russian in Moscow Univ., *Uchenyîa Zapiski*, Otd. Fiz.-Mat., v. 21, 1904, p. 1–121.

Extracts from text

EDITORIAL NOTE: Miss HUCKEL has acknowledged to us that her statement regarding what is tabulated in Tables 3–4 is incorrect, that what she really has tabulated is $-(2/\beta k)dY_k/d\tau$, as suggested by Dr. J. C. P. MILLER in *Math. Reviews*, v. 10, May 1949, p. 329.

In the Report of GARRICK & KAPLAN there are a number of Tables including a 5D table of Y_k for k from -5 to $+5$ and with M as parameter varying from .1 to 2, and with corresponding values of τ.

677[L].—A. N. LOWAN & WILLIAM HORENSTEIN, *On the Function* $H(m, a, x) = \exp(-ix)F(m+1-ia, 2m+2; ix)$. NBS, *Mathematical Table*, no. MT 19. Second ed., June 15, 1949. Washington, D. C., ii, 20 p. 14.9 × 23.2 cm.

This table, previously reviewed in *MTAC*, v. 1, p. 156, is here reprinted, with permission of the editors of *Jn. Math. Phys.* No changes were made in the text or tabular material.

678[L].—WILHELM MAGNUS & FRITZ OBERHETTINGER, *Formulas and Theorems for the Special Functions of Mathematical Physics. Translated from the German by* JOHN WERMER. *Published and distributed in the Public Interest by authority of the Attorney General under License No. A-1341.* New York, Chelsea Publ. Co., 231 West 29th St., 1949, viii, 172 p. $3.50; 15% discount to teachers and members of the Amer. Math. Soc. on direct purchase from the publishers. Offset print.

The first German edition of this valuable work (1943), from which this English translation was made, was reviewed in *MTAC*, v. 3, p. 103–105. Since we there gave the detailed

table of contents it will not be necessary to repeat them here. Most of the errata which we then listed have been corrected in this translation, but those noted below still remain. The translation and original correspond page for page throughout.

But we reviewed also the second enlarged and greatly improved German edition (1948), in *MTAC*, v. 3, p. 368–369, which may be procured comparatively easily. Hence many people who might have purchased the English edition may now prefer the better German work.

The errata referred to above are the following:

P. 24, l. 13, *for* u^3, *read* w^3.
P. 46, l. 13, *before* $=0$, *insert* y.
P. 73, exponents μ, of lines 3 and 5, are badly placed.
P. 83, l. 11, *for* $\alpha\gamma$ are real and $\alpha + 1 > \gamma > 1$, *read* α and γ are real, and $\gamma > 0, \alpha > \gamma - 1$; l. 14, *delete*.
P. 88, l. -1, *for* 0 ± 1, *read* $0, \pm 1$.
P. 108, l. 3, *for* x[3 times], *read* t.
P. 119, heading, *for* §2, *read* §1; col. 2, l. 5, *for* $y|$, read $|y|$.
P. 131, l. -1, for the image function of the original function $t^\nu J_\mu(at)$ in the Laplace transform table, the second edition has

$$p^{-\mu-\nu-1}\frac{\Gamma(\mu+\nu+1)}{2^\mu\Gamma(\mu+1)}\,a^\mu{}_2F_1[\tfrac{1}{2}(\mu+\nu+1),\ \tfrac{1}{2}(\mu+\nu+2);\ \mu+1;\ -a^2/p^2],$$

not

$$(-1)^m\frac{\Gamma(\mu+\nu+1)}{(p^2+a^2)^{\frac{1}{2}(\nu+1)}}\,P_\nu^{-\mu}(p/\sqrt{p^2+a^2})\mathrm{Re}(\nu+m) > -1.$$

R. C. A.

679[L].—NBSCL, *Tables of the Confluent Hypergeometric Function* $F(\frac{1}{2}n, \frac{1}{2}; x)$ *and Related Functions*. (NBS, *Applied Mathematics Series*, no. 3.) Washington, D. C., 1949, xxii, 73 p. 19.8 × 26.1 cm. For sale by the Superintendent of Documents, Washington, D. C., 35 cents. See *MTAC*, v. 3, p. 414, 483.

The confluent hypergeometric function $F(\alpha, \gamma; x)$ is defined by

$$F(\alpha,\gamma;x) = \sum_{j=0}^{\infty}\Gamma(\gamma)\Gamma(\alpha+j)x^j/[\Gamma(\alpha)\Gamma(\gamma+j)j!]. \tag{1}$$

It is easily verified that it is the solution of the differential equation

$$xy'' + (\gamma - x)y' - \alpha y = 0 \tag{2}$$

satisfying the initial conditions $y(0) = 1$ and $y'(0) = \alpha/\gamma$. KUMMER's formula

$$F(\alpha,\gamma;x) = e^xF(\gamma-\alpha,\gamma;-x) \tag{3}$$

is a direct consequence of this property and may be verified by substitution into (2). In this review we use the abbreviation $F_n = F(\frac{1}{2}n, \frac{1}{2}; x)$. Using the series expansion (1) for the right side in (3) it is readily seen that when n is a positive odd integer, then F_n equals e^x multiplied by a polynomial of degree $\frac{1}{2}(n-1)$. In particular

$$F_1 = e^x, \qquad F_3 = e^x(1+2x), \qquad F_5 = e^x(1+4x+\tfrac{4}{3}x^2). \tag{4}$$

The ratios $g_n = F_n/F_{n-2}$ satisfy the recurrence relation

$$g_{n+2} = (2x+2n-1)/n - (n-1)/ng_n. \tag{5}$$

Formulae (4) and (5) permit to calculate successively all $F_n(x)$ for odd n, and this method was used in the preparation of the Tables. Various other relations were used to check the accuracy of the tables, and it is believed that the error is less than one unit of the last decimal place.

In the present tables the subscript n of F_n is always an odd integer. The original objective was the tabulation of $\ln F_n$, but it turned out that the function $(2nx)^{-\frac{1}{2}}\ln F_n$ is more

convenient for interpolation and has the added advantage of being of the order of magnitude of unity, over a considerable range for n and x. The main part of the tables (p. 14–73) is devoted to the tabulation of this function for $n = 3(2)201$ and $x = .1(.01).6(.1)2(.2)7(1)45(5)100$. For $n = 3(2)21$, the range .6 to 2 is covered in steps of .05. Rows correspond to values of x, columns to n. On p. 2–6 the function F_n itself is tabulated for $x = 0(.01).1$ and $n = 3(2)201$, and on p. 8–11 we find $F_n/\cosh\{(2n-1)x\}^{\frac{1}{2}}$ for the same range of x and $n = 43(2)201$. The latter table is given for interpolation purposes since in the range covered F_n varies very rapidly. All tables are to 6D, except the table of F_n on p. 3–6 which is to 7S.

A pleasing feature of the volume is a comparatively large collection of formulae and asymptotic expansions for F_n and certain related functions. (Most of these formulae are not directly connected with the tables.) An interpolation chart shows how many terms of the bivariate Lagrange interpolation formula are required for various n and x in order to obtain an accuracy to 6D.

Confluent hypergeometric functions appear in many applications. The present tables were designed especially for statisticians. Their primary purpose is in connection with the sequential t-test designed by A. WALD. It is also of great use in the analysis-of-variance and the ordinary t-test.

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680[L, M, S].—C. J. BOUWKAMP, "On the effective length of a linear transmitting antenna," *Philips Research Reports*, Eindhoven, Holland, v. 4, June 1949, p. 179–188. 15.4 × 23.4 cm.

$F_1(H) = \int_0^H J_1(x)\cos x\,dx/x$, $F_2(H) = \int_0^H J_2(x)\sin x\,dx/x$, $C(4H) = \int_0^{4H}(1-\cos x)\,dx/x$, $H_{e,s} = 2F_1(H)\sin H$, $H_{e,a} = 2F_2(H)\cos H$, $R = 30[C(4H) - \sin^2 2H]$, $R_s \approx 80|H_{e,s}|^2$, $R_a \approx 64|H_{e,a}|^2$.

In Table I are 6D values of $F_1(H)$, $F_2(H)$, $C(4H)$ for $H = 0(.1)3$.

In Table II are given 4D values of $H_{e,s}$, $H_{e,a}$, R_s, R_a, $R_s + R_a$, R, for $H = .1(.1)3$.

Extracts from text

681[L, M].—HAROLD LEVINE & JULIAN SCHWINGER, "On the theory of diffraction by an aperture in an infinite plane screen. I," *Phys. Rev.*, s. 2, v. 74, 1948, p. 958–974. 19.8 × 26.6 cm.

On p. 972–974, $\int_0^1 v^{-n-m}(1-v^2)^{\frac{1}{2}}J_{n+\frac{1}{2}}(\alpha v)J_{m+\frac{1}{2}}(\alpha v)\,dv$ is evaluated for $m = 1, 2$, and $n = 1, 2$.

682[L, S].—J. MEIXNER, *Lamé's Wave Functions of the Ellipsoid of Revolution*. Translated by MARY L. MAHLER from "Die Laméschen Wellenfunktionen des Drehellipsoids," Zentrale für wissenschaftliches Berichtswesen der Luftfahrtforschung des Generalluftzeugmeisters (ZWB), Berlin-Adlershof, *Forschungsbericht* no. 1952, June, 1944, for the National Advisory Committee for Aeronautics. *Technical Memorandum*, no. 1224, Washington, D. C., April 1949, iv, 102 p. 19.8 × 26.3 cm.

The basic differential equation treated is

$$\frac{d}{d\eta}\left[(1-\eta^2)\frac{df}{d\eta}\right] + \left(\frac{-\mu^2}{1-\eta^2} - k^2c^2\eta^2 + \lambda\right)f = 0. \tag{1}$$

Corresponding to given parameters μ and $\gamma = kc$, there exists a countably infinite set of eigenvalues $\lambda_r^\mu(\gamma)$ corresponding to which (1) has a solution, called the eigenfunction, which can be represented by

$$X_r^{\mu(1)}(\xi;\gamma) = \sum_{r=-\infty}^{\infty} i^r a_{r,r}^\mu(\gamma) P_{r+r}^\mu(\xi), \tag{2}$$

where $P^{m}_{\mu+r}(\xi)$ is the associated LEGENDRE function of the first kind and r is always even. It should be noted that if in (1) we set $f = (1-\eta^2)^{\frac{1}{2}m}w$, we obtain

$$(3) \qquad (1-\eta^2)w'' - 2(\mu+1)\eta w' + [-\mu(\mu+1) - k^2c^2\eta^2 + \lambda]w = 0.$$

If μ is an integer m, (3) yields the *prolate* spheroidal wave functions (*oblate* if $\gamma = iC$, C real). Fairly extensive tabulations of both the eigenvalues and coefficients proportional to $a^{m}_{n,r}$ for integral m, are available.[3] In fact, the author refers to this text, and his mathematical development closely follows it—at least so far as the determination of the eigenvalues, and the derivation of the relations existing between the various solutions of (2). The author notes that if μ in (1) is equal to $-\frac{1}{2}$, we obtain MATHIEU'S equation (this was already noted.[3]) Although the mathematical treatment applies to general values of μ, the tabular material is restricted to integral values of μ, which are designated by m, as in 3. Before describing the tables, it will be necessary to define a second solution of (1):

$$X^{m(2)}_{n}(\xi;\gamma) = \sum_{r=-\infty}^{r'} i^r a^{m}_{n,r}(\gamma) P^{m}_{n+r}(\xi) + \sum_{r=r'+2}^{\infty} i^r a^{m}_{n,r}(\gamma) Q^{m}_{n+r}(\gamma).$$

In the above $Q^{m}_{n+r}(\xi)$ is the associated Legendre function of the second kind, and r' is some finite integer.

It may be shown that

$$X^{-m(1)}_{n}(\xi;\gamma) = \sum_{r=-\infty}^{\infty} i^r b^{m}_{n,r}(\gamma) P^{-m}_{n+r}(\xi),$$

where

$$b^{m}_{n,r}(\gamma) = [(n+m+r)!(n-m)!/(n+r-m)!(n+m)!]\, a^{m}_{n,r}(\gamma) = a^{-m}_{n,r}(\gamma).$$

The tables comprise the following:

Table 1. Coefficients of $\gamma^2, \gamma^4, \gamma^6, \gamma^8$, and γ^{10} in the power-series expansion of $\lambda_n{}^m(\gamma)$, $m = 0(1)9$, $n = m(1)9$; 10D.

Tables 2, 3, 4, and 5. Coefficients of γ^2, γ^4 and γ^6 in the power-series expansion of $a^{m}_{n,r}/a^{m}_{n,0}$ of (2), $m = 0(1)9$, $n = m(1)9$, $\pm r = [2, 4, 6; 10D]$. (The coefficient of γ^{2k} is zero if $k < |r|$).

Table 6. The coefficients of γ^2, γ^4 and γ^6 in the expansion of $\alpha^{m}_{n,r}/\alpha^{m}_{n,0}$; $m = 0(1)2$, $n = m(1)(5-m)$; $r = [-6(2)-2; 10D]$.

Tables 7, 8, 9, 10. The coefficients of γ^2, γ^4, and γ^6 in the power-series expansion of $b^{m}_{n,r}/b^{m}_{n,0}$; $m = 1(1)9$, $n = m(1)9$, $\pm r = [2(2)6; 10D]$. (For $m = 0$, $b_{n,r} = a_{n,r}$).

The author points out that these truncated power series are useful only for relatively small values of γ, when n is of low order, but that more accuracy is obtainable for high values of n.

In addition to the above tabular material, the author inserted some of BOUWKAMP'S tables of eigenvalues[1] for $m = 0$, and values of the corresponding eigenfunction or its derivative at $\xi = 0$ and 1. There are seven tables of this character, comprising seven pages. Corrections which Bouwkamp made after publication were apparently not known to Meixner; as a result the last two places of these tables are totally unreliable, and more serious errors also remain uncorrected. It should be noted that nearly all values of 1 which Meixner used[5] were corrected and amplified[2] (see review[4]). Since 2 is relatively accessible, Meixner's reproduction of Bouwkamp's erroneous values should be discarded; it is unfortunate that the translator did not replace these seven pages with the more useful tables.[2]

Meixner's new results are contained in Tables 1–10; they duplicate some results of 1 only for $m = 0$. Since Meixner gives no *tabular* values for the eigenvalues or coefficients, his results do not duplicate the tables in 3 but rather add to them. Some asymptotic formulae for the eigenvalues and functions are also given, and there is a graph of $\lambda_n(\gamma)$ for $m = 0$, $n = 0(1)9$. The text contains a valuable compilation of theoretical results, with some discussion of their application to various problems.

The NACA performed a genuinely useful service in making this work available in English.

GERTRUDE BLANCH

NBSINA

[1] C. J. BOUWKAMP, *Theoretische en numerieke Behandeling van de Buiging door een ronde Opening*. Diss. Groningen, 1941

[2] C. J. BOUWKAMP, "On spheroidal wave functions of order zero," *Jn. Math. Physics*, v. 26, 1947, p. 79–92.

[3] J. A. STRATTON, P. M. MORSE, L. J. CHU, & R. A. HUTNER. *Elliptic Cylinder and Spheroidal Wave Functions*. New York, 1941. See *MTAC*, v. 1, p. 157–160.

[4] *MTAC*, v. 3, 1948, p. 99–101; review of items 1 and 2.

[5] The values corresponding to $\gamma^2 = 15$, 20, 25, 50, and 100 are the only ones not reproduced in 2.

683[M].—ERIK HALLÉN, *Iterated Sine and Cosine Integrals*, Roy. Inst. Techn., Stockholm, *Trans.*, no. 12, 1947, 6 p. 17.6 × 24.7 cm.

In antenna theory there are some integral functions which might also be of interest elsewhere. They derive from sine and cosine integrals in the same manner as these derive from sine and cosine. The following are tabulated:

$$L_{11}(x) = C_{11}(x) + iS_{11}(x) = \int_0^x t^{-1}L(t)dt,$$

$$L_{21}(x) = C_{21}(x) + iS_{11}(x) = \int_0^x t^{-1}e^{it}[L(2t) - L(t)]dt,$$

$x = [0(.2)14; 5D]$, where $L(x) = C(x) + iS(x) = \int_0^x t^{-1}(1 - e^{-it})dt$, $C(x) = \gamma + \ln x - Ci(x)$ and $S(x) = Si(x)$. There is also a table of $L_{11}(x)$, for $x = [14.2(.2)28; 5D]$. The tables were computed by mechanical integration. The fifth decimal is uncertain within two units.

Extracts from text

EDITORIAL NOTE: For a less extended 6D table of L_{11}, by C. J. BOUWKAMP, see *MTAC*, v. 3, p. 303–304.

684[M].—J. LE CAINE, *A Table of Integrals involving the functions* $E_n(x) = \int_1^\infty e^{-xu}u^{-n}du$. National Res. Council, Div. of Atomic Energy, *Report* no. MT 131. 1948, i, 45 leaves. 20.3 × 27.4 cm.

The definite and indefinite integrals involve products of two or more factors $E_n(kx)$, $E_n(ax + b)$, $E_n(k|x - y|)$, e^{kx}, $e^{-k|x-y|}x^m$. In the single integrations x is the integration variable, and x and y in the double integrations. For numerical tables of such integrals see *MTAC*, v. 2, p. 260, 272 and v. 3, p. 303–304, and RMT 683.

R. C. A.

685[M, Q].—H. C. VAN DE HULST, "Scattering in a planetary atmosphere," *Astrophys. Jn.*, v. 107, 1948, p. 220–246. 16.9 × 24 cm.

In this paper the problem of multiple scattering in a planetary atmosphere, both with and without a diffusely reflecting bottom surface, is discussed. We assume that the atmospheric scattering is isotropic with an albedo a, and that the ground surface reflects the radiation according to Lambert's law, with an albedo b. The mathematical functions occurring in the calculations are of three types. **T. 1**: for $E_n(x) = \int_1^\infty t^{-n}e^{-xt}dt$, $E_1^{(2)}(x) = \int_x^\infty t^{-1}E_1dt$ the functions tabulated are $E_1(x)$, $E_1^{(2)}(x)$, $E_1^{(0)}(x) = e^{-x}$, $E_2(x)$, and also $\ln x + \gamma$ (γ being Euler's constant) are each tabulated to 6D; and $2E_2(x)$ to 5D. **T. 2–3**: For $F_n(b, x) = \int_0^x e^{bt}E_n(t)dt$, there are 4D tables for $F_n(-s, x)$, $s = 10, 5, 2, 1$; and of $e^{-sx}F_n(s, x)$, $s = 0, 1, 2, 5, 10$, in each case $n = 1, 2$. **T. 4**: For $G_{mn}(x) = \int_0^x E_m(t)E_n(t)dt$, $G'_{mn}(x) = \int_0^x E_m(t)E_n(x - t)dt$, there are 5D values for $m, n = 1, 2$; also there are 4D values for

$g(x) = 2 - 2E_3(x) - G_{12}(x) - G_{12}'(x)$ and $c(x) = [x - \frac{1}{2} + E_3(x)]/x$. In each of the tables x = .01, .02, .05, .1(.1).4(.2)1(.5)3, ∞.

Extracts from text

EDITORIAL NOTE: For considerably more extensive tables of $E_n(x)$ see *MTAC*, v. 2, p. 272.

686[N].—FINANCIAL PUBLISHING COMPANY, **A.** *Eighth Coupon Rate Monthly Bond Values showing accurate monthly values on bonds and other redeemable securities paying interest semi-annually. For eighth coupon rates only from* $1\frac{1}{8}\%$ *to* $3\frac{7}{8}\%$. Boston, Financial Publishing Co.; London, Routledge & Kegan Paul Ltd., 1949 [the printing and binding were not completed until Feb. 1949], x, 537 p. 25 × 36.8 cm. Bound in red imitation leather, thumb index. $125.00. In the back of the volume is a loose, slightly smaller (24.5 × 36 cm.) paper covered reprint, 48 p., dated 1948, from the second edition (p. 1109–1154, 0% coupon) of *Monthly Bond Values*, 1941, and entitled: *Tables of Present Worth* of 100, *from* 1 *month to* 110 *years*.

B. *Executive Bond Values Tables. Desk Edition, Showing net returns on Bonds and other redeemable Securities paying Interest semi-annually.* Boston, Financial Publishing Co.; London, George Routledge & Sons, Ltd., 1947, 1984 p. 11 × 18.6 cm. Bound in leather, full gilt, thumb index. $25.00.

A. In the 8 years since the second edition of *Monthly Bond Values* was published (see *MTAC*, v. 1, p. 114–115), continuing low interest rates and the development of sale at competitive bidding of corporate bonds have produced a number of issues with $\frac{1}{8}\%$ coupons which were not covered by complete tables in the second edition. Hence the need for this new volume which is essentially supplemental to the second edition.

The 12 coupon rates here are $1\frac{1}{8}\%(\frac{1}{4}\%)3\frac{1}{8}\%$. The yields are from 0 to 5.70%, at interval .05%, as before. Maturities, as in the earlier volume, progress by monthly intervals up to 40 years 3 months, from $40\frac{1}{4}$ years to 50 years semi-annually, 50 years to 65 years annually, and 70 years to 110 years by 5 years. All tabulation is to 6D, adjusted from at least 8D computation.

As formerly, on every page are 12 columns (one for each month) and 115 lines corresponding to 0 (.05%) 5.70%, the various yields (in black face type on each side of the page), making 1380 entries all arranged in groups of five, admirably spaced for ready use. Also in black face type are given under the appropriate columns the amounts of accrued interest on $100. for 1, 2, 3, 4, 5 months. Pages 536–537 contain a 6D table of accrued interest on $100. for any number of days 1(1)180, in 6 months, for each of the coupon rates. An 8D "Table of factors" is provided (p. 532–534) for computing values for 1 to 30 days, between the even months, for each of the yields.

In the foreword to the volume are explained various methods of using and applying the tables which are largely a reprint of the second edition. Part of what was said in review of the earlier volume is applicable to the present book, and the comments made at that time apply also to this volume.

The manner of preparing the tables was almost wholly dictated by trade practice and by what would be most intelligible to bond dealers. The method of computation involved the addition of the constant difference between values at successive coupon rates. This constant difference was computed to a minimum of two decimal places beyond the six decimal places contained in the table, and in most cases to more than two extra decimals. The mathematician notes with interest that trade practice calls for the addition to every invoice of simple interest at the coupon rate. Hence it became necessary to compute the theoretically correct "flat" value and from this to subtract simple accrued interest. Thus when an invoice is made for bonds sold at a yield basis the sum which changes hands will be the theoretically correct amount, but it will be itemized in two distinct elements neither

of which may be considered mathematically exact. This feature is most readily observed in the case of any bond selling to yield its coupon rate. The value is shown as 100 on coupon dates only, and for intermediate dates at something less than 100; but if the accrued interest at the bottom of the column be added a correct result will be obtained.

Another curious feature is that in spite of solicitude for correct semi-annual compound interest in the body of the book, values for less than six months' maturity are shown at true discount, in accord with trade custom of dealers and banks. Again, the editors believed that labelling a present-worth table as the value of a 0% bond evokes a clearer concept to the average bond dealer, than a proper labelling would.

Without a volume of this character, dealing in municipal securities would be a somewhat difficult process. For example, a dealer offers a 3½% coupon municipal having 20 years and 4 months to run on a 1.95% yield basis. What dollar price must the purchaser pay? Turning to the 3½% coupon table, find the page "20 years, column 4 months, and on the margin the yield 1.95%," and the price is given as 125.913848 or $1259.14 per $1000 bond. Also at the bottom of the column is given the accrued interest in monthly intervals, $.583333 per $100 or $5.83 per $1000 bond. The methods of adjusting for fractional parts of a month, and for many other special cases of practice, are explained clearly in the introductory pages. These pages contain also quite explicit instructions involving the use of adding or accounting machines where these are available.

These 1941 and 1949 tables are the most extensive of their kind, and are of value particularly to dealers and those handling state and municipal securities where price is usually quoted on a yield basis, also for insurance companies where exact computations of yield are essential. The tables are obviously of importance also to the actuary or accountant in making up amortization schedules.

B. Since the 1941 and 1949 volumes are too large and bulky for use outside an office, two small volumes have earlier found considerable favor. One of these, *Comprehensive Bond Values*, 1936, had the advantage of containing tables for odd eighth coupons, now being increasingly used. The other, *Pocket Edition of Monthly Bond Values*, 1939, had the advantage of monthly maturity intervals but with no eighth coupons. The volume under review contains a combination of the material in these volumes.

It covers coupon rates ½%(⅛%)4%(⅛%)5%. The maturities for ¼% coupons are monthly to 20 years, quarterly to 30 years, semi-annually to 50 years, annually to 56 years, bi-annually 56 years to 60 years, and at 5 year intervals from 60 years to 100 years. On the ⅛% coupons maturities are shown monthly to 10 years, semi-annually to 42 years, annually to 56 years, bi-annually to 60 years, and at 5 year intervals to 100 years. Coupon rates for ½% and ¾% stop at 28 years. For ⅛%, ⅞%, 1⅛% and 1⅜% they stop at 30 years. The tables are in condensed forms and show values to 2D only.

In the foreword to the book are descriptions of methods of use of the tables and at the end are tables of factors to be used on callable bonds and tables of current yields on bonds or stocks, given the price and coupon or dividend rate. There are also included tables for accrued interest on a 365 day basis from ⅛% (⅛%) 4% (⅛%) 6% for use with government securities. The volume is very valuable for the convenience of salesmen and others in determining quick approximate figures.

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687[R].—ITALY, ISTITUTO GEOGRAFICO MILITARE, Florence, **A.** [GIOVANNI BOAGA], *Sulla Rappresentazione Conforme di Gauss*, 1941, ii, 65 p. **B.** [GIOVANNI BOAGA], *Sulla Rappresentazione Conforme di Gauss, Allegati*, (i) *Parte I, Tavole Numerici*, 1942, vii, 25 p.; (ii) *Parte II, Tavole Ausiliarie . . .*, 1942, xiv, 49 + *Errata Corrige* 4 p.; (iii) *Parte III, Applicazioni*, 1942, 38 p. 15.5 × 21.6 cm.

This work treats the conformal representation of the earth considered as an ellipsoid of revolution onto the Euclidean plane. The tabular material is for that part of the representa-

tion which lies between latitudes 36° and 48°, the range in which Italy lies. The main treatise gives a theoretical development of the representation with special attention to the preparation of formulae well adapted to calculation.

Let λ be the longitude in seconds of a given point P on the ellipsoid, measured from a convenient meridian, and let B be the length of arc on the meridian from the equator to a reference point. Next let x, y be the rectangular coordinates of the point P' in the plane corresponding to the point P. Then one has approximately

$$\log (x - B) = 2 \log \lambda + T_0 + C_2\lambda^2$$

$$\log y = \log \lambda + T_1 + C_3\lambda^3$$

where the quantities T_0, T_1, C_2, C_3 depend on the latitude of the point P. This dependence is tabulated in Part I. More precisely the quantities B, T_0 and T_1 and their differences per second together with C_2, C_3, and their logarithms are given for every minute of latitude between 36° and 48°. This table is based on the HAYFORD ellipsoid. B is given to a millimeter, the T's to 7D and the C's to 4S or 5S.

Part II is based on the BESSEL ellipsoid and gives slightly different values for the above quantities. Both Parts I, II give three other coefficients used to compute the "convergence of the meridian," the "modulus of linear deformation" and "angular reduction." These are also tabulated for each minute of latitude. The use of the tables is illustrated in Part III.

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688[U].—GEORGE G. HOEHNE, *Practical Celestial Air Navigation Tables. Volume II, Latitudes 20° to 39° North inclusive.* Miami, The Navigation Publishing Co., 1943, 256 p. 15 × 24.3 cm. $3.65.

This volume was reviewed earlier in *MTAC* v. 2, p. 129–131; the copyright difficulties which formerly prevented its sale to the public have been taken care of and it is now generally available at the price indicated above. In this connection, it may be pointed out that many of the altitudes in the volume were taken directly from the British ANT (or our H. O. 218 which was a direct copy of it); during the war, both of these sets of tables were restricted. With the closing of the war, these restrictions were lifted and the way was open for arrangements to meet British Crown copyright requirements.

It is hoped that another volume, similar to this but covering latitudes 40°–59° North, will soon be available.

CHARLES H. SMILEY

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689[U].—O. A. DE AZEREDO RODRIGUES, *Gráficos Usados na Navegação. D. N. 7–1.* Rio de Janeiro, Diretoria de Hidrografia e Navegação, 1948, 48 p. 16.5 × 24.5 cm. 35 cruzeiros.

This volume contains twenty nomograms useful in the ordinary computations of navigation. The author is a Capitão-de-Corveta in the Brazilian Navy; he already has one other navigation table to his credit, *Tábuas para Retas de Altura* (RMT 516, *MTAC*, v. 3, p. 114–115). The nomograms are well designed for practical use and are clearly printed on good heavy white paper. The volume is bound in light green cardboard and presents a neat appearance.

Following a brief preface and a list of abbreviations, there is an index. The remainder of the volume is given over to the nomograms and to explanations of the use of a few of the more complicated ones. For each one except the last, one or more examples are given with the correct answers.

The first nomogram provides the difference of latitude, dL, and departure, P, for a rhumb line, given the distance, $D = 0(1')60'(2')150'(5')220'$, and the course angle, $C = (0\text{–}90°)$. The corresponding equations are $dL = D \cos C$, $P = D \sin C$.

The second converts departure, $P = 0(1')100'$ into difference of longitude, $dLo = 0 - 220'$, given the mean latitude, $L_m = 0 - 90°$. $dLo = P \sec L_m$.

The third provides the distance, $D = 0 - 100$ nautical miles to an object of known elevation, $E = 0 - 2400$ meters, given the vertical angle $\alpha = 0(5')6°$ and the height of eye of the observer, 1–45 meters. A small table converts height of eye into dip of the horizon, $\beta = 2'(1')12'$. The basic equation is $D = kE \cot(\alpha - \beta)$ where k is the fraction of a nautical mile equal to one meter. The fourth yields the distance, $D = 100 - 3500$ meters, given the elevation $E = 0(1)52$ meters of an object and the vertical angle $\alpha = 0°30'(5')3°30'$ subtended by it. $D = E \cot \alpha$.

The fifth gives the distance, $D = 3 - 1000$ meters travelled in $S = 1 - 60$ seconds at a velocity, $v = 4(0.5)36$ knots. $D = kvs$ where k = number of meters per second in one knot.

The sixth yields the distance $D = 12$ to 880 nautical miles travelled in $H = 2^h(10^m)$-$6^h(20^m)24^h$ at a speed $V = 6(0.5)36$ knots. $D = VH$.

Number 7 provides the correction $\gamma = 0(0°.1)13°.5$ to be applied to a radio bearing before plotting it on a Mercator map, having given the difference in longitude $dLo = 0(0°.5)31°$ and the mean latitude $L_m = 0(0°.5)65°$. The corresponding equation is $\gamma = \frac{1}{2} dLo \sin L_m$.

The eighth nomogram is somewhat larger than the remaining charts; it requires two facing pages, which are in reality a single sheet folded in the middle. Even though it has been placed in the middle of the book and special care has been taken in binding it, the crease will probably cause some trouble until the volume is limp from use. This nomogram allows one to find the azimuth angle $A = 16°$ to 164° of a celestial body when the local hour angle $t = 5°$ to 170°, the declination $d = 75°$ same name $-25°$ opposite name to latitude and the latitude $L = 0$ to 70° are given. The equation solved by the nomogram is

$$\cot A \sec L = \tan d \csc t - \tan L \cot t.$$

The PAGEL coefficient, cot A sec L, may thus be evaluated with this nomogram.

Number 9 gives the azimuth angle $A = 0 - 180°$ as a function of local hour angle, $t = 0$ to 180°, declination, $d = 0(1°)90°$, and altitude $h = 0(1°)90°$. The equation here is $\csc A = \csc t \sec d \cos a$.

Nomograms 10 and 11 provide respectively the altitude, $h = 0(1°)90°$ and local hour angle $t = 0$ to 90° of a celestial body when it crosses the prime vertical. The arguments are in both cases latitude and declination; in the altitude nomogram, $L = 1°$ to 90°, $d = 1°$ to 90° and in the local hour angle chart, $L = 0(1°)90°$ and $d = 0(1°)60°$. The equations are $\csc h = \csc d \sin L$ and $\sec t = \cot d \tan L$.

Nomograms 12 and 13 give respectively the altitude, $h = 0(1°)90°$ and the local hour angle $t = 10°(1°)90°$ of a body at maximum azimuth angle. The arguments in the former nomogram are $L = 1°(1°)90°$ and $d = 1°(1°)90°$; in the latter, they are $L = 0(1°)60°$ and $d = 0(1°)90°$. The equations are $\csc h = \csc L \sin d$ and $\sec t = \tan d \cot L$.

Nomograms 14 and 15 yield the azimuth angle $A = 0 - 180°$ and local hour angle $t = 5°$ to 175° of a body which is rising (or setting). The arguments are respectively $L = 0$ to 89°, $d = 1°$ to 90° and $L = 0(1°)90°$, $d = 0(1°)60°$. The equations are $\sec A = \csc d \cos L$ and $\sec t = \cot d \cot L$.

Nomogram 16 provides the correction $c = 0(1')80'$ to be applied to a circum-meridian altitude, given the local hour angle $t = 0(15')17°30'$ and the declination $d = 60°$ same name to 30° opposite name h, and latitude L with the same range as d.

Nomogram 17 gives the change in altitude Δh per minute of arc change in local hour angle Δt. With $L = 0$ to 90° and azimuth angle 0 to 90°, $\Delta h/\Delta t = 0(0'.05)1'$. The equation used is $\Delta h/\Delta t = \sin A \cos L$.

Nomogram 18 yields the change in altitude $\Delta h = 0(1')30'$ corresponding to a change in latitude $\Delta L = 0(1')30'$ and an azimuth angle of 0 to 180°. $\Delta h = \Delta L \cos A$.

Nomogram 19 is a simple nomogram for multiplication and division. Chart no. 20 is intended for plotting lines of position and obtaining fixes. On it the scales of distance and latitude vary with the mean latitude, while the scale of longitude is constant. This is in contrast to the usual charts of this type in which the scale of longitude varies and the other two scales are equal and constant; there is much to recommend this approach.

It should be pointed out that where the smallest division of a scale in the nomogram is uniform, the interval of the corresponding argument is indicated; otherwise the range of values covered by the scale is noted. It will be clear that the accuracy of a given nomogram may often be increased by interpolating between the marks provided.

CHARLES H. SMILEY

690[Ù].—D. H. SADLER, "Tables for astronomical polar navigation," Institute of Navigation, *Jn.*, v. 2, 1949, p. 9–24. 15.5 × 24.9 cm.

Summary: The special features of polar navigation are examined with a view to the design of tables for astronomical navigation in polar regions. The use of long intercepts and curved position lines is thoroughly investigated. It is shown that considerable economy of presentation can be achieved; as an example, a one-page table covering all bodies and both polar caps is given. Making the fullest use of polar astronomy, samples are given of permanent tables for the sun and stars independent of the *Air Almanac*.

MATHEMATICAL TABLES—ERRATA

References to Errata have been made in RMT **659** (Dale), **662** (Briggs), **665** (Boll), **666** (Lehmer), **672** (Feller), **676** (Huckel), **678** (Magnus & Oberhettinger), **682** (Bouwkamp, Meixner), **683** (Hallén); N **106** (Watson), **107** (FMR, Gauss), **109** (DeMorgan, Rheticus).

160.—D. H. LEHMER, *Guide to Tables in the Theory of Numbers*, 1941.

P. 14, table, col. 3, the second and third entries from the bottom should be interchanged.

P. 15, l. 9, *for* Creak 1, *read* Creak 2.

P. 47, l. −9, *for* 195 [twice], *read* 295.

P. 74, l. 18, *for* $29 \leq p$, *read* $17 \leq p$.

P. 76, l. 7, *for* VORONO" *read* VORONOĬ.

P. 95, l. 17, *for Haupt Exponents, read Haupt-Exponents.*

P. 102, Glaisher 16, l. 2, *for* 127, *read* 125.

P. 109, l. 10–11, *for milia accuratis, read millia accuratius.*

P. 114, in reference Ostrogradsky 1 *for* v. 1, 1838, *read* v. 3, 1836.

P. 129, the error for $n = 1019681$, occurs in Burckhardt 1, *not* 2. The error for $n = 2012603$ occurs in Burckhardt 2, *not* 3. *Delete* l. 4 (446021) and 5 (446023) since these errata were listed by Burckhardt himself, v. 1, p. IV.

P. 134, l. −6, *for* 61·330413, *read* 1399·14407.

P. 134, Cunningham 28, p. 243, col. 11, l. 18, *read* 9683, *instead of* 4683; p. 244, in tables giving elements y, of primes $(y^2 + 1)/13$, omit the entry 671, and *for* 3930 in col. 4, l. 6, *read* 2930.

P. 139, Euler 2_3, 193 factor of $81^3 + 1$ *not* of $82^3 + 1$; $1068^3 + 1 = 5^6 \cdot 73$, *not* $5^6 \cdot 73$; 773 factor of $1090^3 + 1$, *not* of $1080^3 + 1$.

P. 156, Kraitchik 4[f_1] *for* $p = 116537$, *read* 116337.

P. 174–6, the following items in the index are slightly out of alphabetical order: Forms, quartic; Ince; Logarithms; Pierce; Sum of powers.

A few careful readers have pointed out the carelessness of the first definition of the function $\psi(n)$ on p. 7. This function has only half the value attributed to it by this definition whenever n is divisible by 8.

D. H. L.

161.—D. N. LEHMER, "A complete census of 4 × 4 magic squares," Amer. Math. Soc., *Bull.*, v. 39, 1933, p. 764–767.

This paper contains a list, p. 767, of the number of normalized squares with various first columns and last rows.

For (15, 2, 1) $(10, 5, 3)_{12}$, $(9, 6, 3)_{11}$, $(8, 7, 3)_2$, $(8, 6, 4)_3$, *read* (15, 2, 1) $(10, 5, 3)_{13}$, $(9, 6, 3)_{12}$, $(8, 7, 3)_3$, $(8, 6, 4)_4$; *for* (14, 3, 1) $(8, 6, 4)_2$, *read* (14, 3, 1) $(8, 6, 4)_3$; *for* (12, 4, 2) $(11, 6, 1)_4$, *read* (12, 4, 2) $(11, 6, 1)_5$. These corrections change the total number of 4×4 magic squares from 539136 to 549504.

S. URATA, A. SAKAI & A. HIRAYAMA

Yamagata-Ken
Japan

UNPUBLISHED MATHEMATICAL TABLES

82[F].—L. POLETTI, *Factor Table and List of Primes for the 30000 natural numbers nearest 15,000,000.* Manuscript table deposited in the library of the American Math. Soc. New York.

This table gives new information for the range 14984970–15000000. The second half from 15000000 to 15015000, is also covered by W. P. Durfee's factor table for the 16th million, a table which is in the same library.

The factor table, which the author calls "Neocribrum," is a "type 3 table" arranged in the usual way modulo 30. On p. 1 are given data on the distribution of the primes in this range. Thus there are 1809 primes which are also classified modulo 30. There are 159 prime pairs. There are 113 consecutive composite numbers following 14996687.

Poletti is the author of *Tavole di Numeri Primi entro Limiti Diversi e Tavole Affini*, Milan, 1920.

D. H. L.

83[G, I].—H. E. SALZER, *Coefficients of the first fifteen General Laguerre Polynomials.* Ms. in possession of the author.

The writer announced previously (*MTAC*, v. 2, p. 89) a manuscript giving the coefficients of LAGUERRE polynomials, which are a special case of general Laguerre polynomials $L_n^{(\alpha)}(x)$, namely for $\alpha = 0$. The present manuscript gives the polynomials in α which are the coefficients of x^ν in the general Laguerre polynomial

$$L_n^{(\alpha)}(x) = e^x x^{-\alpha} \frac{1}{n!} \left(\frac{d}{dx}\right)^n (e^{-x} x^{n+\alpha}) = \sum_{\nu=0}^{n} \binom{n+\alpha}{n-\nu} \frac{(-x)^\nu}{\nu!}, \text{ for } \nu = 0(1)n,$$

and for $n = 0(1)15$.

H. E. SALZER

AUTOMATIC COMPUTING MACHINERY

Edited by the Staff of the Machine Development Laboratory of the National Bureau of Standards. Correspondence regarding the Section should be directed to Dr. E. W. CANNON, 418 South Building, National Bureau of Standards, Washington 25, D. C.

TECHNICAL DEVELOPMENTS

Our contribution under this heading, appearing earlier in this issue, is "The California Institute of Technology Electric Analogue Computer" by Prof. G. D. MCCANN.

DISCUSSIONS

Procedure for the Machine or Numerical Solution of Ordinary Linear Differential Equations for Two-Point Linear Boundary Values

Introduction. Increased attention is being focused on machine and numerical solutions of differential equations which cannot be solved by ordinary mathematical methods. There is need for more information on this

subject for engineers and others who deal with such equations. This paper describes a procedure applicable to numerical or machine solutions of the general non-homogeneous ordinary linear differential equation with variable coefficients where the form of these coefficients does not easily permit of solution by series. The method is based on the well-known properties of linear differential equations.

Ordinary differential equations of order higher than the first commonly describe problems where the known boundary conditions are expressed at two different values of the independent variable. Such problems are known as two-point boundary-value problems. Although a great many linear equations, such as the BESSEL and LEGENDRE equations, may be rigorously handled by the method of FROBENIUS, there are frequently those where the variable coefficients of the derivatives are so complex that a series solution is not feasible. For such equations, recourse to solution by numerical methods or by some type of computing machine or analyzer may be sought. In this event, a difficulty is at once encountered if divided boundary conditions are present. In the case of an equation of order n, the dependent variable and its $n - 1$ derivatives must possess assigned values at some point within the interval in order that a machine or step-by-step solution may proceed from that point. Consider the case of a second-order equation where the two boundary conditions are divided between both ends of the range of the independent variable. Only one of an unlimited number of possible values of the initially unknown dependent variable or one of its derivatives, as the case may be, at one end of the interval of solution will satisfy the boundary condition at the other end. For a fourth-order equation with equally divided boundary conditions, a double latitude of possible initial choices would exist.

The theory of ordinary linear differential equations appears extensively in the mathematical literature.[1, 2, 3] Application to the two-point boundary problem where numerical or machine solutions are involved does not appear to be generally well known. The purpose of this paper is to show in relatively simple mathematical terms and by graphical illustration how the two-point boundary problem may be handled. The method applies to any ordinary linear differential equation of order n where the boundary conditions are expressible in terms of linear combinations of the dependent variable and its $n - 1$ derivatives and where K boundary conditions are known at one point of the interval of solution and the remaining $n - K$ conditions are known at some other point of the interval. It is shown that for $K \leq n/2$ only $K + 1$ trial solutions with arbitrarily selected values for those derivatives which are initially unknown are required to determine the unique solution satisfying all n boundary conditions. It is also shown that $n + 1$ trial solutions will give data for any solution of the non-homogeneous equation. In addition, a possible graphical procedure is suggested for converging on the solution of non-linear equations.

Theory. Consider an ordinary non-homogeneous linear differential equation of order n with variable coefficients. This is

$$(1) \quad f_0(d^n y/dx^n) + f_1(d^{n-1}y/dx^{n-1}) + \cdots + f_{n-1}(dy/dx) + f_n y = f.$$

Assume $f_0, f_1, \cdots f_n, f$ are finite, one-valued, and continuous functions of x in the interval $a_0 \leq x \leq b_0$, and that f_0 does not vanish at any point in the interval. Under these conditions, there is known to exist a solution y such

that y and its first n derivatives are continuous and have unique values at every point in the interval. The reduced or homogeneous equation corresponding to equation (1) is

$$(2) \qquad f_0(d^ny/dx^n) + f_1(d^{n-1}y/dx^{n-1}) + \cdots + f_{n-1}(dy/dx) + f_ny = 0$$

and is known to have n and only n linearly independent solutions,[4] $y_1, y_2, \cdots y_n$. The known complete solution of equation (1) is then

$$(3) \qquad y = c_1y_1 + c_2y_2 + \cdots + c_ny_n + y_p = y_p + \sum_{i=1}^{i=n} c_iy_i,$$

where y_p is any particular solution of equation (1) and $c_1, c_2, \cdots c_n$ are arbitrary constants to be determined by the n boundary conditions.

If the quantities B_k which assume boundary values are linear,[5] we may express them in the following manner:

$$(4) \qquad B_k = \sum_{j=0}^{j=n-1} a_{k,j}(d^jy/dx^j), \qquad k = 1, 2, \cdots n,$$

where each of the coefficients, $a_{k,j}$, represents a constant or some known function of x, and where $|a_{k,j}| \neq 0$ since the boundary conditions are linearly independent. Substituting the general solution, equation (3), into equation (4) and factoring out the c's, we have for the particular point, $x = x_k$, at which B_k has a known value,

$$\left(B_k = \sum_{i=1}^{i=n} c_i \sum_{j=0}^{j=n-1} a_{k,j}(d^jy_i/dx^j) + \sum_{j=0}^{j=n-1} a_{k,j}(d^jy_p/dx^j) \right)_{x=x_k}, \qquad k = 1, 2, 3, \cdots n.$$

In this expression for B_k, all terms are of fixed value. Since there are n such linear expressions for the B's, and the product $|a_{k,j}| \cdot |d^jy_i/dx^j|$ does not vanish, we may solve explicitly for the n values of the c's. Carrying this out one obtains linear equations of the form,

$$(5) \qquad c_i = b_i + \sum_{k=1}^{k=n} b_{i,k}B_k, \qquad i = 1, 2, \cdots n.$$

The term b_i is a combination of the constant values of y_p and its derivatives for the particular x involved, and $b_{i,k}$ is a combination of the constants $a_{k,j}$ and the fixed values of y_i and its derivatives. Substituting equation (5) into equation (3) one obtains

$$(6) \qquad y = y_p + \sum_{i=1}^{i=n} b_iy_i + \sum_{k=1}^{k=n} B_k \sum_{i=1}^{i=n} b_{i,k}y_i.$$

This is a formal statement of the fact that the general solution may be expressed directly in terms of linear combinations of the n boundary parameters. It now becomes evident that for a given equation of order n, if n arbitrary but linearly independent solutions y_i of equation (2) and any particular solution y_p of equation (1) are obtained, then the solution to the problem with any desired values of divided boundary parameters, $B_1, B_2, \cdots, B_n$, known at any points within the interval (a_0, b_0), may be obtained by direct substitution into equation (6). It should be emphasized that y_i and y_p are *any* solutions to their respective equations, barring linearly dependent

y_i's, without restriction on initial or final values. It is seen from equation (6) that, if the boundary conditions are all homogeneous, then each B equals zero and the solution is simply

$$y = y_p + \sum_{i=1}^{i=n} b_i y_i.$$

Also, if the equation is homogeneous but not all of the boundary conditions are homogeneous, then

$$y = \sum_{k=1}^{k=n} B_k \sum_{i=1}^{i=n} b_{i,k} y_i.$$

The term in $b_i y_i$ is zero since b_i is a linear combination of the fixed boundary values of y_p and its $n - 1$ derivatives, and these quantities exist only in the non-homogeneous equation. If the equation and boundary conditions are all homogeneous, then it is apparent that $y = 0$, and there is no problem.

The advantage of expressing the solution directly in terms of the boundary parameters will now be illustrated for a fourth-order equation with boundary values B_1 and B_2 existing at $x = a$ and B_3 and B_4 existing at $x = b$. The limits a, b are ordinary points of the interval (a_0, b_0). For $n = 4$, the solution y and its first $n - 1$ derivatives may be written from equation (6) as

$$y = B_1 Y_1 + B_2 Y_2 + B_3 Y_3 + B_4 Y_4 + Y$$
$$dy/dx = B_1(dY_1/dx) + \text{etc.}, \qquad d^2y/dx^2 = B_1(d^2Y_1/dx^2) + \text{etc.},$$
$$d^3y/dx^3 = B_1(d^3Y_1/dx^3) + \text{etc.},$$

where Y_1, Y_2, Y_3, Y_4 are linear combinations of y_1, y_2, y_3, y_4, and Y is a linear combination of the same y's and y_p. Now consider the problem of a machine or step-by-step solution starting at $x = a$ where the two boundary parameters have the desired values $B_1 = \beta_1$ and $B_2 = \beta_2$. From the two boundary relations at $x = a$ given by equation (4) we may write

$$\beta_1 = [a_{1,0}y + a_{1,1}(dy/dx) + a_{1,2}(d^2y/dx^2) + a_{1,3}(d^3y/dx^3)]_{x=a}$$
$$\beta_2 = [a_{2,0}y + a_{2,1}(dy/dx) + a_{2,2}(d^2y/dx^2) + a_{2,3}(d^3y/dx^3)]_{x=a}$$

from which two of the initial values of the derivatives may be solved in terms of the remaining two. Thus, for any arbitrary values assigned to $y_{x=a}$ and $(dy/dx)_{x=a}$, the values of the second and third derivatives at $x = a$ may be calculated.[6] With B_1 and B_2 assigned the values β_1 and β_2, respectively, the solution and its first derivative become

$$y = B_3 Y_3 + B_4 Y_4 + Y_5, \qquad dy/dx = B_3 dY_3/dx + B_4 dY_4/dx + dY_5/dx,$$

where Y_5 is the new function, $Y + \beta_1 Y_1 + \beta_2 Y_2$. At $x = a$, Y_j and dY_j/dx (where $j = 3, 4, 5$) assume fixed values so that

$$(7) \qquad y_{x=a} = d_3 B_3 + d_4 B_4 + d_5, \qquad (dy/dx)_{x=a} = e_3 B_3 + e_4 B_4 + e_5,$$

where the d's and e's are constants. Equations (7) show that the initial values of y and dy/dx are related linearly to the boundary parameters at $x = b$. This linearity may be represented graphically as shown in Fig. 1. Although the existence of these linear families of boundary parameters is now established, their determination is still unknown for any trial solution.

Assume now that the problem at hand requires that $B_3 = \beta_3$ and $B_4 = \beta_4$ at $x = b$. It is readily seen that there exists a point S which defines the proper initial choice of $y_{x=a}$ and $(dy/dx)_{x=a}$. It is also evident from the linearity of equations (7) that, in proceeding on a straight line joining any two points, such as 1 and 2 in Fig. 1, the values of B_3 and B_4 will vary linearly with the distance measured along that line. With these facts in mind, let us make three separate trial solutions with any arbitrary combination of $y_{x=a}$ and $(dy/dx)_{x=a}$ which define three non-collinear points 1, 2, 3. The initial values of the second and third derivatives for each trial are of course determined so as to satisfy $B_1 = \beta_1$ and $B_2 = \beta_2$ at $x = a$. When these trial solutions have reached $x = b$, the values of y and its three derivatives at this point are used to calculate B_3 and B_4. By linear interpolation point P_1 which represents a point where B_3 equals the desired value β_3 may be located on line 1–2. Likewise points P_2 and P_3 which lie on the desired β_3 line are located by linear interpolation and extrapolation along lines 2–3 and 3–1. Similarly points Q_1, Q_2, Q_3 are located on the desired β_4 line. These two parameter lines may now be constructed and the desired solution point S determined by their intersection.

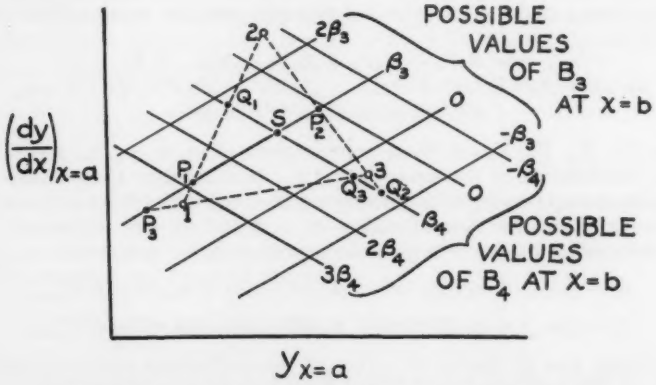

FIG. 1. Control plot for solution of fourth-order linear differential equation with equally divided boundary values.

Although the desired boundary conditions can now be satisfied with a fourth solution which begins with the correct combination of $y_{x=a}$ and $(dy/dx)_{x=a}$, it is not necessary to effect this fourth solution. For any particular value of x in the interval (a, b), the correct values of y and its derivatives may be obtained by linear interpolation and extrapolation to the solution point S from the corresponding values at points 1, 2, 3. This further linearity is at once evident from equations (7) if we replace B_3 and B_4 by any two of the derivatives of y at this particular value of x.

Consider next a sixth-order equation with three boundary conditions known at $x = a$ and the other three at $x = b$. Here, there is a triple latitude of possible choices of the three initially unknown values of y and its five derivatives at $x = a$ and only one unique combination will satisfy the given

conditions at $x = b$. To solve this problem, we may visualize, in place of Fig. 1, three families of parallel planes in a three-dimensional space defined by the three initial values of y and its derivatives which are unknown. To determine the solution point S representing the common intersection of the three boundary-value planes will require four different trial solutions, each satisfying the three conditions at $x = a$. These trials will define four non-coplanar points in the three-dimensional space. Linear interpolation and extrapolation along any three non-coplanar lines joining these points will determine three sets of three points, each set uniquely defining one of the desired boundary-value planes. Again the values of y and its derivatives can be obtained at point S for any value of x by a three-dimensional linear interpolation and extrapolation from the four points.

In the case of a second-order equation with its two boundary conditions divided, only two trials, each satisfying the initial condition at $x = a$, are necessary to establish the solution. The relationship is shown graphically in Fig. 2 where the solution point S is determined by the linearity along the line joining the trial points 1 and 2.

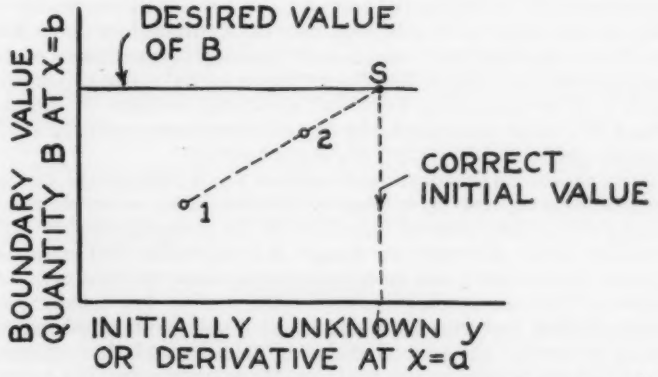

FIG. 2. Control plot for solution of second order linear differential equation with equally divided boundary values.

So far, only cases of equally divided boundary conditions have been discussed. If unequal division occurs, the solution should be started at the point where the greater number of known conditions exists. In the case of a fourth-order equation where three conditions are known for $x = a$, only one derivative of y is unknown, and two trials will be sufficient for solution.

Boundary conditions need not exist at the extremities of the interval of solution. In such cases the solution may begin at a point within the interval where at least half of the conditions are known. It will be necessary to reverse the direction of the independent variable in order to cover the complete range, but otherwise the procedure will be the same.

The following rules may be stated from the foregoing development. They hold for any ordinary linear differential equation of order n defined by equation (1) where K boundary conditions are known at one point within

the interval of solution and the remaining $n - K$ conditions are known at some other point within the interval. If $K \leq n/2$, only $K + 1$ trial solutions with $K + 1$ arbitrarily selected initial values of the initially unknown derivatives are required to determine uniquely the desired solution which satisfies all boundary conditions. If data for the solution for any or all possible combinations of the n boundary conditions are required, then any n solutions, barring linearly dependent ones, of the homogeneous equation without reference to initial conditions and any one solution of the complete equation, or a total of $n + 1$ arbitrary trial solutions, must be made.

Discussion. Although the $n + 1$ trial solutions are sufficient to solve all boundary value problems for a given equation, in most practical cases, where interest is centered on a given set of boundary values, the method described using $K + 1$ trials, where $K \leq n/2$, will be the simpler procedure. Even in this case $K + 1$ families of solutions may be had with the $K + 1$ trials, since any boundary value at $x = b$ for each of the K boundary parameters and any value of b within the interval (a_0, b_0) may be used. All of these solutions must satisfy the same conditions at $x = a$.

Graphical illustration has been used to describe the linearities involved in order to aid in visualizing the problem. The solution point and the values of the various functions at this point may be determined by direct solution of the linear relationships involved. And indeed, if it is necessary to discuss an equation of order higher than the sixth, our spatial visualization which is limited to three dimensions would not aid in this problem. If the method of the $n + 1$ trials is adopted, the use of determinants will facilitate the necessary computation of b_i and $b_{i,k}$ in equation (6).

There are several practical considerations which place some limits on the success of these methods. It is essential that the distances between the trial points be of the same order of magnitude as the distance from any point to the solution point. Although the danger of interpolation and extrapolation on curved lines is absent, still the errors inherent in any machine or numerical solution will limit the extent of accurate extrapolation. It is usually possible in most physical problems by approximation, comparison, and reasonable guessing to predict the general region of solution and thus choose trial points which are not too far removed from the solution point. If a reasonably close estimate cannot be made, any $K + 1$ trials will point to the approximate location of S whereupon $K + 1$ additional trials in the neighborhood of S will yield the solution.

In order to insure sufficient accuracy in the results, a reasonable estimate of the range of magnitudes of y and its $n - 1$ derivatives must be made in the case of a machine solution. It may be necessary to repeat one or more trials with adjusted scale factors if the estimate is far in error.

Although $K + 1$ trial solutions are mathematically sufficient for solution, one or two more may be desirable in the case of a machine solution as a check on the accuracy of the work. In the case of the fourth-order equation with equally divided boundary values, the triangular configuration of trial points shown in Fig. 1 might well be replaced by four points representing the corners of a square. Any three of the six possible line segments joining the four points may be considered as locating these points. These three segments then determine independently three points on the desired

parameter line. The remaining three segments may be considered as dependent on the first three and hence will determine three dependent points on the same parameter line.

The method described in this paper has been used successfully by the author in the solution of a problem in the theory of shells involving a fourth-order equation with equally divided boundary conditions. Triangular sets of trial points in the machine solution used were adequate.

In the case of non-linear equations or linear equations where the boundary conditions are non-linear a procedure similar to that described in the foregoing paragraphs is suggested. In this case graphical representation would be indicated, and, in a control plot corresponding to Fig. 1, the families of lines representing boundary parameters would no longer be straight lines or linearly spaced. However, by the principle of uniqueness of solution, it is evident that any one boundary parameter line of one family will not cross any other boundary parameter line of the other family more than once. Also, if a reasonably good initial estimate is made, it should be possible to converge on the solution with a few successive sets of trial points.

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[1] E. L. INCE, *Ordinary Differential Equations*. New York, 1944.
[2] A. R. FORSYTH, *A Treatise on Differential Equations*. Sixth ed. London, 1933.
[3] I. S. & E. S. SOKOLNIKOFF, *Higher Mathematics for Engineers and Physicists*. New York, 1941.
[4] The criterion for linear independence of these solutions is that the Wronskian of the y_i's (where $i = 1, 2, 3, \cdots n$) and their $n - 1$ derivatives does not vanish.
[5] Boundary values are said to be linear if they can be expressed as linear combinations of the dependent variable and its $n - 1$ derivatives.
[6] If in some actual problem $y_{x=a}$ and $(dy/dx)_{x=a}$, for example, took on assigned boundary values, then these may not be changed. Thus $(d^2y/dx^2)_{x=a}$ and $(d^3y/dx^3)_{x=a}$ would be the unknown initial quantities.

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The automatic message accounting equipment is able to "read" perfectly 80 coded digits per second. Paper tape, on which records of thousands of telephone calls are stored, contains information on the calling and called numbers, the month, day, and exact time to tenths of minutes at which a telephone conversation begins and ends. The tapes are run through "reader" machines, and the information is assembled, translated, sorted, summarized, and printed so that telephone bills may be made from it.

2. ANON., "Automatic computer for aircraft-motion mathematics," *Instruments*, v. 22, n. 1, Jan. 1949, p. 69, 70, illustr. 20 × 26.5 cm.

3. ANON., "GE analyzer for Indian Institute," *Electrical Engineering*, v. 68, 1949, p. 84. 29 × 21.25 cm.

An a-c network analyzer, for installation at the Indian Institute of Science, Bangalore.

4. ANON., "Instruments guide industry," *Business Week*, n. 995, Sept. 25, 1948, p. 70–74, illustr. 25.5 × 28.25 cm.

5. DAVID L. ARENBERG, "Ultrasonic solid delay lines," Acoustical Soc. Amer., *Jn.*, v. 20, 1948, p. 1–26, bibl., diags., tables. 19 × 27 cm.

Abstract: The development of an ultrasonic delay line for the storage of pulses one microsecond long for periods up to 2000 microseconds is discussed. Theoretical considerations of the piezoelectric transducer involving the quartz crystals and acoustical constants of the medium developed by EMSLIE, HUNTINGTON, and BENFIELD are applied to determine the band width and power loss of the delay line. Problems, peculiar to the use of solids because of the presence of more than one mode of propagation and the distribution of energy between the different modes on reflection from an air surface, are evaluated for a wide range of POISSON ratios. Measurements of velocity and attenuation in some polycrystalline metals, single crystals, and glasses are given. Fused quartz was found to be the most promising material.

Delay lines constructed using a light source and photoelastic pick-up were not completely developed, but several types using crystal as well as transmitter pick-up were satisfactory. For delay times longer than 300 microseconds, multiple reflection paths in two and three dimensions were used and their preparation from fused quartz discussed.

6. A. R. BOOTHROYD & E. COLIN CHERRY, "Analogue calculating machine for functions of a complex variable," *Nature*, v. 163, April 30, 1949, p. 687. 17.4 × 25.4 cm.

7. J. G. BRAINERD & T. K. SHARPLESS, "The ENIAC," *Electrical Engineering*, v. 67, n. 2, Feb. 1948, p. 168–172, diags. 21.5 × 29 cm.

8. J. A. BRONZO & H. G. COHEN, "A note on analog computer design," *Rev. Sci. Instrs.*, v. 20, n. 2, Feb. 1949, p. 101–102. 19 × 26.5 cm.

A method is proposed for the simplification of computers designed to solve certain types of partial differential equations.

9. C. CHU, "The recent development in the electronic digital variable computer," Chinese Institute of Engineers, *Jn.*, v. 6, no. 1, May 1948, p. 58–65. 14.5 × 23 cm.

10. HARRY M. DAVIS, "Mathematical machines," *Scientific American*, v. 180, April 1949, p. 28–39, illustrs. 21.25 × 29 cm.

This article is principally concerned with the mathematical machines which count, i.e., the digital machines, and a detailed description of the digital idea is presented. It emphasizes the fact that, although these machines are powerless to do anything creative, they are extremely useful in that they are not subject to human intervention and are capable of enormous speed. Current large-scale computer developments are mentioned including: (1) the IBM Selective Sequence Calculator and the electronic Type 604 Calculator, which is ten times faster than the previous IBM commercial calculating punches; (2) the Moore School EDVAC, having a far more capacious memory and greater versatility than its forerunner, the ENIAC; (3) the twin computers known as BINAC and the UNIVAC, developed by John Mauchly and J. P. ECKERT, JR.; (4) the colossal computing device which is being designed at the Institute for Advanced Study for purposes of weather prediction and possibly for work on questions of atomic physics and economic statistics; (5) the Raytheon Manufacturing Company computer; (6) the Harvard Mark III computer; (7) the MIT "Project Whirlwind" for the construction of a computer for military purposes; and (8) advanced computers now under construction by four British Laboratories. Every current computer will soon become outdated in this new age of electronic thinking; however, the older machines will not become obsolete, because problems exist suitable for every type to

handle. Also the relay, as a computing device, and the much speedier electron tube is discussed with a detailed explanation of the operation of the 20 ENIAC accumulators and of the binary representation of numbers. The application of the binary system to logic as well as numbers is stressed. In particular the author mentions the various memory devices now being developed and outlines future applications to the 1950 census, adjustment of maps, engineering computations, hydrodynamics, large-scale planning in the Air Force, etc.

11. HERMAN H. GOLDSTINE & JOHN VON NEUMANN, *Planning and Coding of Problems for an Electronic Computing Instrument.* Part II, v. 3. Princeton, N. J. Institute for Advanced Study, 1948, iii + 23 p. 21.6 × 27.9 cm.

This volume deals with the manner of incorporating subroutines into a complete problem. After considerable explanation, the authors conclude that this is best accomplished for small subroutines by recording them into the main routine as needed. For larger subroutines a preparatory routine automatically modifies the addresses of the subroutine relating to positions within itself by adding a quantity corresponding to the difference between the actual location of the subroutine in the memory and the assumed location at the time the subroutine was initially coded. There is a detailed discussion of suitable preparatory routines for a single subroutine and for a group of subroutines all of which are considered as coding problems. The volume closes with a description of proposed details for handling main routines and subroutines and the preparatory routine for machine input.

While the subject matter is of appreciable importance and the manner of handling it shows ingenuity, there is some question as to whether its treatment in this report is optimum. The following are some of the points one might question:

1. In order to use the method described, the coding of subroutines must be handicapped somewhat by certain limitations. For example, it is not permissible to use one half of a word for numerical storage and the other half as an order. It is also not permissible to build up addresses by the addition of numerical quantities, some of which occupy address positions, unless these quantities satisfy certain requirements. While such limitations are simple in character, their existence lends some importance to use of other methods which impose fewer restrictions to achieve the same result.

In addition to other methods which utilize auxiliary input equipment (for example, that suggested by SAMUEL LUBKIN in "Proposed programing for the EDVAC," Aberdeen Proving Ground, Jan. 1947), we may mention one suggested by ALAN L. LEINER and RALPH J. SLUTZ of the Electronic Computer Section of NBS. It is based upon the provision of extra binary digits in the coding as an indication of whether the corresponding address is to be taken as an absolute location in the memory or as "relative" to the memory location in which it occurs. Also, there is a method based upon the provision of one or more "base numbers" for automatically modifying appropriate classes of addresses (suggested by Samuel Lubkin primarily for utilization in connection with iterations but incidentally of value in connection with the present problem). In addition, it is possible to use a different criterion than that chosen by the authors for determining which addresses are to be modified. For example, NBSMDL has recently coded an analogous preparatory routine for the EDVAC, using the first binary digit in each address to indicate whether or not the address requires modification.

2. Time requirements for machine modification of subroutines are certainly negligible compared to those for manual operations relating to insertion of subroutines from a "library" into the memory. This does not, however, necessarily rule out special routine-preparing equipment, since the latter may facilitate the manual operations themselves and may make coding more convenient.

3. There seems to be little reason for not including the constants of the preparatory routine in the main routine instead of inserting each constant manually as indicated by the authors. Also, the space taken up by the preparatory routine might be used by the main

routine at some time after the address modification is completed. For this purpose, it might be more convenient to locate the preparatory routine at the beginning of the memory rather than at the end. After reading in the subroutines and the preparatory routine (with constants), the latter could be carried out and followed by instructions to read in the balance of the main routine, re-using memory locations originally containing the preparatory routine.

4. The authors mention the possible convention of having all subroutines coded as if they begin at 0, but they do not require this in their preparatory routine. Such a convention seems simpler to use than the requirement that the actual position occupied by a subroutine be further along in the memory than the position for which it is coded. Use of the convention would cut the length of the preparatory routine. It might also be noted that one of the subroutines (by choice, the longest) can always be inserted in the memory in the same position for which it was originally coded and, in such a case, requires no modification.

5. The coding given by the authors for the preparatory routine can be materially shortened. A new code of the routine has been prepared by Otto T. Steiner & Samuel Lubkin requiring only $36 + 2I$ "words" (where I represents the number of subroutines) as compared to the $58 + 4I$ "words" required in the original routine. It was prepared without the use of flow diagrams or sub-sectioning. Indeed, most MDL coders find such aids, in the form used by the authors, confusing and even detrimental.

Samuel Lubkin

NBSMDL

12. D. R. Hartree, "A historical survey of digital computing machines"; M. H. A. Newman, "General principles of the design of all-purpose computing machines"; M. V. Wilkes, "The design of a practical high-speed computing machine. The EDSAC"; F. C. Williams, "A cathode-ray tube digit store"; J. H. Wilkinson, "The automatic computing engine at the National Physical Laboratory"; A. D. Booth, "Recent computer projects," R. Soc. London, *Proc.*, v. 195A, Dec. 1948, p. 265–287 + 3 plates. Discussion held 4 Mar. 1948. 17.2 × 25.6 cm. Reported in *Nature*, v. 161, May 8, 1948, p. 712–713. 15.5 × 25.5 cm.

The discussion was confined to automatic digital general-purpose computing machines. The EDSAC (Electronic Delay Storage Automatic Calculator) is at the University Mathematical Laboratory, Cambridge, England. Compare *MTAC*, v. 3, p. 214–215.

13. Wm. A. McCool, *An Improved Electronic Analog Computing Circuit.* Report no. P-3423. Washington, Naval Res. Lab., Feb. 23, 1949, 15 p. illustr. 26.5 × 20 cm.

Abstract: An improved electronic analog computing circuit, employing only two vacuum tubes and characterized by both simplicity and accuracy of computation, is described and analyzed. The theoretical error is calculated for integration, differentiation, and constant multiplication with step-function and sine-wave forms of excitation. The results of a cursory experiment, made to evaluate the performance of the improved circuit functioning as a differentiator, agree with the theoretical results.

14. T. K. Sharpless, "Design of mercury delay lines," *Electronics*, v. 20, Nov. 1947, p. 134–138, illustrs. 28.25 × 20.25 cm.

Mercury is used to obtain millisecond delays for radar, computers, and memory devices because it transmits compression waves relatively slowly, introduces negligible loss, and has an impedance comparable to that of crystal transducers. Recirculation and temperature compensation techniques are presented.

15. NORBERT WIENER, *Cybernetics*. Hermann et Cie, Paris, and John Wiley & Sons, New York, 1948, 194 p. 22.5 × 14.5 cm.

The term, cybernetics, has been chosen by Wiener and his collaborators to designate the entire field of control and communication in the animal and the machine. The justification for a common treatment of apparently diverse phenomena in control and communication, ranging, for example, from the operation of purely mechanical feed-back systems to neurological phenomena like memory and visual perception, would appear to lie in the identity of the mathematical approach. However, as is explained in a thirty-nine page narrative which serves as an introduction to the text of the book, the underlying motif of research in cybernetics is rather the investigation of boundary regions of science, areas formerly "neglected as a no-man's land between the various established fields."

The volume, *Cybernetics*, contains eight formal chapters, beginning with a chapter on Newtonian and Bergsonian time and ending with a chapter on information, language, and society. The remaining chapters are concerned, respectively, with "Groups and statistical mechanics," "Time series, information, and communications," "Feed-back and oscillation," "Computing machines and the nervous system," "Gestalt and universals," and "Cybernetics and psychopathology." In addition there is a note, refreshingly objective by comparison with popular press notices on the subject, in which the capability of a computing machine at chess playing is discussed briefly.

The introduction gives a somewhat detailed account of the origin of Wiener's interest in the investigation of the nature of control and communication in the human being and of the manner in which group effort was developed in the field. It is stated that the book represents the outcome of a program of work undertaken jointly with Dr. ARTURO ROSENBLUETH, formerly of the Harvard Medical School, and now of the Instituto Nacional de Cardiologia of Mexico. The assistance of scientists, distinguished for their outstanding work in such diverse fields as electro-mechanics, anthropology, sociology, and statistical opinion sampling, is acknowledged.

In chapter I, "Newtonian and Bergsonian time," the implications of reversibility of time are discussed. It is pointed out that planned physical experiments introduce asymmetry into reversible time systems. In fact, the very nature of the questions asked, of probability and prediction, must of necessity be asymmetrical, distinguishing between the past held in the present by the fixing of certain quantities, and the future. Newtonian astronomy and meteorology, Newtonian dynamics and Gibbsian statistical mechanics, physics, and biology are discussed from the standpoint of reversibility of time. The difference between the irreversible time of evolution and biology and the reversible time of physics was emphasized by Bergson. In Wiener's opinion, the controversy between vitalism and mechanism had at its kernel the fact that Newtonian physics is not the proper frame for biology.

Chapter II, "Groups and statistical mechanics," is, in a sense, an introduction to the following chapter, "Time series, information, and communications." These two chapters alone, while they are exacting from the standpoint of the mathematical maturity and background required of the reader, would appear to justify the cost of publishing the book. For example, a lucid yet concise account is presented of the foundations of statistical mechanics, including the role of LEBESGUE measure and the metrical invariants of transformation groups in the development of the ergodic theory, and a discussion of the notion of entropy. Also a satisfying formal exposition of a statistical theory of time series is made showing how the theory applies to message-noise problems in communication, to the ensemble of time series associated with Brownian motion, and to prediction problems. Single and multiple time series, both continuous and discrete, are treated from the viewpoint of full knowledge of the parts of the series. The role of the theory of time series in quantum mechanics is touched upon and the line of development of a practicable theory of time series involving conclusions based on a sampling of the past is indicated.

In the remaining chapters of *Cybernetics*, IV, "Feed-back and oscillation," V, "Computing machines and the nervous system," VI, "Gestalt and universals," VII, "Cybernetics

and psychopathology," and VIII, "Information, language, and society," the reader of moderate mathematical interest will find the going easier. It is necessary, says the author, to use mathematical symbolism and mathematical techniques in IV, but the mathematical treatment is made more palatable to the average reader by extensive verbal explanation. In IV, feed-backs found in human and animal reflexes and homostasis are discussed. In chapters V, VI, and VII, discussion of both normal and abnormal action of the nervous system centers about the ultra-fast, automatically-sequenced computing machine as a model. The reviewer cannot repress a feeling that the author is unrealistic in his discussion of possible future lines of development of such computing devices, for example, the incorporation in them of the ability to learn; nevertheless, the utility of such devices outside the field of numerical computation is made clear.

The last chapter, "Information, language, and society," ends the *Cybernetics* on a sombre note, seeming to possess a tincture of misanthropy. One can only hope that in its impact upon sociology, the new science of cybernetics will exert a beneficent influence and that the social sciences will not be too greatly retarded in development by the comparatively close coupling of the observer with phenomena mentioned in this chapter.

E. W. CANNON

NBS

EDITORIAL NOTE: Professor Wiener's book was printed in France and is filled with typographical errors, sometimes making the mathematics quite unintelligible.

NEWS

Association for Computing Machinery.—CHARLES B. TOMPKINS has resigned from the Council of the Association, and JAN RAJCHMAN has been appointed. At present, the membership totals about 600. Prof. EDY VELANDER of Sweden has been authorized to undertake the formation of a Swedish section, and Mr. ALBERT CAHN of NBSINA is in charge of forming a California section of the Association.

About 200 members of the Association attended the meeting in Oak Ridge, Tennessee, April 18–20, 1949. Those who were present found it to be an informative and satisfying meeting. The Association greatly appreciated the welcome and hospitality of the Oak Ridge National Laboratory, the Oak Ridge Institute of Nuclear Studies, and the other Oak Ridge groups.

The papers listed below were delivered. Plans are being made to furnish summaries (or the full version) of these papers to members. In some cases the full paper will be submitted to the editors of *MTAC*.

1. "Address of welcome," A. M. WEINBERG, Oak Ridge Nat. Lab. 2. "Limitations of electronic digital computing machines," E. W. CANNON, NBS. 3. "Analog and special-purpose computing machines," S. H. CALDWELL, MIT. 4. "Inversion of large matrices," GERTRUDE BLANCH, NBSINA. 5. "Problems in linear programing," EMIL SCHELL, Office of the Air Comptroller, USAF, Washington. 6. "Computing machinery in Australia," T. G. ROOM, Univ. of Sidney, Australia. 7. "The forgotten man of computing," MINA REES, ONR, Washington. 8. "Statistical theory of round-off errors," H. D. HUSKEY, NBSINA. 9. "The relations of symbolic logic and large-scale calculating machines," EDMUND C. BERKELEY, New York. 10. "Calculation of neutron age by Monte Carlo," A. S. HOUSEHOLDER, Oak Ridge Nat. Lab. 11. "Computing statistical atomic fields on the selective sequence electronic calculator," L. H. THOMAS, IBM. 12. "Binary notation in arithmetic algorithms and operation code partitions," B. F. CHEYDLEUR, Naval Ordn. Lab., White Oak, Silver Spring, Md. 13. "602A calculating punch—time saver in computing thermodynamic properties of gases," JACK BELZER, H. L. JOHNSTON, & LYDIA SAVEDOFF, Dept. Chem., Ohio State Univ., Columbus, O. 14. "The register code for the ENIAC," R. F. CLIPPINGER & B. DIMSDALE, BRL, Aberdeen, Md. 15. "Recent progress on the Selectron," JAN RAJCHMAN, Radio Corp. Amer., Princeton, N. J. 16. "Analog computer for FOURIER transforms in crystal analysis," RAYMOND PEPINSKY, Alabama Polytechnic Inst., Auburn, Ala. 17. "The philosophy of computing machine design," GEORGE R. STIBITZ, Burlington,

Vt. 18. "Internal conversion calculation on Mark I," M. E. ROSE, Oak Ridge Nat. Lab., & BERNARD SPINRAD, Argonne Nat. Lab., Chicago. 19. "Experience with the use of the AIKEN Relay Calculator," CLARENCE ROSS, Naval Proving Ground, Dahlgren, Va. 20. "Logical design of direct-coupled vacuum tube control circuits," THEODORE KALIN, Computation Lab., Harvard Univ. 21. "The statistical mechanics of high polymers," GILBERT W. KING, Cambridge, Mass. 22. "The NBS Interim Computer," SAMUEL LUBKIN, NBS. 23. "Uses of pulse transformer in a-c coupling of computers," RALPH SLUTZ, Electronics Div., NBS. 24. "An intercomparison of storage devices," S. N. ALEXANDER, NBS.

NBSINA.—Two symposia were held on June 22 through July 1, 1949, on subjects pertinent to the effective utilization of automatic digital computing machinery. A representative group of the nation's engineers, physicists, and mathematicians participated.

The subject of the first one, June 22–25, was the *Construction and Applications of Conformal Maps*. The applications of conformal maps in such fields as aerodynamics and electronics were emphasized, and special attention was devoted to the actual current needs of research workers in these fields. Construction methods were discussed, along with the problem of programing such methods on existing and proposed automatic digital computing systems. In this connection, particular reference was made to the electronic machine now being designed at NBSINA under the direction of HARRY D. HUSKEY.

The program opened with a one-day course designed to acquaint those attending the Symposium with the preparation of problems for automatic digital computing machines. The following talks were included: "Definition of an automatic digital computing machine" by H. D. Huskey; "Description of a specific automatic computer" by ROSELYN SIEGEL, NBSINA; "Programing the solution of n simultaneous linear equations" by H. D. Huskey; and a laboratory session in which participants were urged to work out detailed routines for causing the computers to perform such operations as division, floating addition, solving a simple differential equation, and possibly solving a set of simultaneous linear equations. On subsequent days the following Sessions were held:

I. June 23, General Session, J. H. CURTISS, NBS, chairman:

"On network methods in conformal mapping computation" by R. VON MISES, Harvard Univ.

"Conformal mapping of domains of higher topological structure illustrated by flow patterns" by R. COURANT, New York Univ.

II. June 23, Session on Physical and Industrial Applications, D. V. WIDDER, Harvard Univ., chairman:

"Applications of conformal mapping to torsional rigidity, principal frequency, and electrostatic capacity" by G. SZEGÖ, Stanford Univ.

"Some industrial applications of conformal mapping" by H. PORITSKY, General Electric Co.

"Conformal maps involving multiple-connected regions and their technical applications" by G. STEIN, Westinghouse Electric Corp.

"On the use of conformal mapping in problems of two-dimensional elasticity" by I. S. SOKOLNIKOFF, UCLA.

III. June 24, Session on Fluid Dynamics, J. L. BARNES, UCLA, chairman:

"On the HELMHOLTZ problem of conformal representation" by A. WEINSTEIN, U. S. Naval Ordn. Lab. and Univ. of Maryland.

"Aspects of conformal mapping in aerodynamics" by I. E. GARRICK, NACA.

"On THEODORSEN's method of conformal mapping" by ALEXANDER OSTROWSKI, Univ. of Basle and NBS.

"On conformal mapping of variable regions" by S. E. WARSCHAWSKI, Univ. of Minnesota and NBS.

"Fluid dynamics, conformal mapping, and numerical methods" by ANDREW VAZSONYI, U. S. Naval Ordn. Test Sta. Pasadena, Cal.

IV. June 24, Session on Theory of Conformal Maps, W. T. MARTIN, MIT and UCLA, chairman:

"Some remarks on variational methods applicable to multiple-connected domains" by D. C. SPENCER, Stanford Univ.

"A variational method for simply-connected domains" by A. C. SCHAEFFER, Purdue Univ.

"Kernel functions and conformal mapping" by STEFAN BERGMAN, Harvard Univ., & MENAHEM SCHIFFER, Stanford Univ.

"The kernel function and canonical conformal maps" by ZEEV NEHARI, Washington Univ.

"A new proof of the Riemann mapping theorem" by P. R. GARABEDIAN, Stanford Univ.

V. June 25, General Session, J. W. GREEN, UCLA, chairman:

"Conformal mapping applied to electromagnetic field problems" by ERNST WEBER, Polytechnic Inst., Brooklyn.

"An extremal method in conformal mapping" by L. V. AHLFORS, Harvard Univ.

"Some generalizations of conformal mappings occurring in gas dynamics" by LIPMAN BERS, Syracuse Univ.

"The use of conformal mapping in the study of flow phenomena at the free surface of an infinite sea" by EUGENE P. COOPER, U. S. Naval Ordnance Test Sta., Pasadena, Cal.

"Recent contributions of the Hungarian school to conformal mapping" by G. SZEGÖ, Stanford Univ.

VI. June 25, Session on Numerical Methods, H. F. BOHNENBLUST, Cal. Inst. Techn., chairman:

"Relaxation methods as ancillary techniques" by Sir RICHARD SOUTHWELL, London, England.

"The use of conformal mapping to compute flows with free streamline" by D. M. YOUNG, Harvard Univ.

"An approximation method for conformal maps" by LEE H. SWINFORD, Univ. of California, Berkeley.

The second Symposium dealt with *Probability Methods in Numerical Analysis*, with special reference to the techniques now known among physicists by the name, "*Monte Carlo.*" It was held on June 29–July 1, under the joint sponsorship of NBSINA and the RAND Corp. with the cooperation of the Oak Ridge Nat. Lab.

A word of explanation concerning the "Monte Carlo" technique is in order here. It is well known in the theory of stochastic processes that the probability distributions associated with certain random walk or random flight problems satisfy certain classical integro-differential equations. This fact provides a novel method for approximate numerical integration of such equations through the medium of building up large samples of trials of corresponding random walks. It also suggests the possibility and desirability of directly constructing appropriate random walks as discrete mathematical models for given physical situations, to be used in place of the more classical continuous models. This method of solution of problems in mathematical physics by sampling techniques based on random walk models constitutes what is known as the "Monte Carlo" method. The method as well as the name for it were apparently first suggested by JOHN VON NEUMANN and S. M. ULAM. "Monte Carlo" techniques are now being used rather extensively in connection with high-speed automatic digital computing machinery.

Many mathematical and physical problems remain to be solved in the theory of "Monte Carlo" techniques. Most of the current "Monte Carlo" applications are classified; however, the proceedings of the Symposium were unclassified. The program was as follows:

I. June 29, Orientation Session, J. H. CURTISS, chairman:

"The Monte Carlo method" by S. ULAM, Los Alamos Sci. Lab.

"Discussion of the Monte Carlo method" by J. VON NEUMANN, Inst. Adv. Study

II. June 29, Session on Physical Applications of Stochastic Methods, J. VON NEUMANN, chairman:

"Introductory remarks" by the chairman

"Showers produced by low energy electrons and photons" by ROBERT R. WILSON, Cornell Univ.

"Nomograms for Monte Carlo solution of the Milne problem" by B. I. SPINRAD, Argonne Nat. Lab. and G. H. GORTZEL, New York Univ.

"Neutron age calculations in water, graphite, and tissue" by A. S. HOUSEHOLDER, Oak Ridge Nat. Lab.

"Methods of probabilities in chains applied to particle transmission through matter" by LEWIS NELSON, Oak Ridge Nat. Lab., & WENDELL DEMARCUS, Fairchild Engine and Airplane Corp.

"Multiple-scattered gamma rays" by UGO FANO, NBS

"Stochastic methods in statistical mechanics" by GILBERT W. KING, Arthur D. Little Co.

III. June 30, Session on Physical Applications of Stochastic Methods (continued), FRANK C. HOYT, Argonne Nat. Lab., chairman:

"Calculations on a water shield for fast neutrons, I" by MARIA MAYER, Argonne Nat. Lab.

"Calculations on a water shield for fast neutrons, II" by PRESTON HAMMER, Los Alamos Sci. Lab.

"A Monte Carlo technique for estimating particle attenuation in bulk matter" by B. A. SHOOR, Northrop Aircraft; LEWIS NELSON, Oak Ridge Nat. Lab.; WENDELL DEMARCUS, & ROBERT L. ECHOLS, Fairchild Engine and Airplane Corp.

"Neutron transmission through thick slabs" by WILLISTON SHOR, U. S. Navy.

"Estimation of particle transmission by random sampling" by HERMAN KAHN, RAND Corp. & T. E. HARRIS, RAND Corp.

IV. JUNE 30, Session on Random Digits, JERZY NEYMAN, Univ. of California, Berkeley, chairman:

"Generation and testing" by GEORGE W. BROWN, RAND Corp.; NICHOLAS METROPOLIS, Los Alamos Sci. Lab.; & GEORGE E. FORSYTHE, NBS.

"Various techniques used in connection with random digits" by J. VON NEUMANN.

General discussion from the floor.

V. JULY 1, Mathematical Session, JOHN W. TUKEY, Princeton Univ., chairman:

"The connection between stochastic processes and partial differential equations" by WILL FELLER, Cornell Univ. and NBSINA; MARK KAC, Cornell Univ. and NBS; & J. L. DOOB, Univ. of Illinois.

Discussion from the floor.

VI. July 1, Round Table Discussion: Critique of Applications and Discussion of Possible New Directions for Research, JOHN W. TUKEY, discussion leader:

Panel: J. NEYMAN; M. A. GIRSCHICK, Stanford Univ.; W. A. WALLIS, Univ. of Chicago; D. H. BLACKWELL, Howard Univ.; T. W. ANDERSON, Columbia Univ.; & L. J. SAVAGE, Univ. of Chicago.

Also on June 27–28, a condensed, relatively elementary course on automatic computation was held at NBSINA. The purpose of this course was to introduce interested persons to the logical theory and performance characteristics of automatic digital calculators being developed at the present time. Preparation of problems for the calculators was emphasized. It was composed of two lecture and two laboratory sessions.

OTHER AIDS TO COMPUTATION

Bibliography Z–IX

1. R. T. Brown, "Notes on the use of the log-log slide rule in connection with trigonometric functions," *Jn. Engin. Educ.*, v. 38, 1948, p. 393–394. 15 × 22.5 cm.

2. A. E. Carter & D. H. Sadler, "The application of the National Accounting Machine to the solution of first-order differential equations," *Quart. Jn. Mech. Appl. Math.*, v. 1, 1948, p. 433–441. 15.4 × 23.4 cm.

Summary: Milne's formula for approximate quadrature is used as the basis of a method for the solution of first-order differential equations on the National machine. The method, which is illustrated by a numerical example, enables the machine to form the required dependent variable without the necessity for conversion from a sum to an integral.

3. Harold P. Knauss, "Slide rule calculations of radioactive decay," *Science*, v. 107, 26 Mar. 1948, p. 324.

Selected sentences: The decay of a radio-active substance is described by the equation $N = N_0 e^{-\lambda t} = N_0 10^{-\lambda' t}$, in which N is the number of particles remaining at the time t; N_0 is the initial number of the particles; λ, the decay constant or fraction of the number present disintegrating per unit of time; and $\lambda' = .4343\lambda$. On the log log rule a single setting of the slide gives the values of N/N_0 for the time. Three significant figures are obtained with a 10″ slide rule.

4. H. W. Richards & D. H. Sadler, "The application of the National Accounting Machine to the calculation of apparent places of stars," RAS, *Mo. Not.*, v. 108, 1948, p. 154–161 + a folding tabular plate. 17 × 25.1 cm.

Summary: A method is developed for the systematic calculation of apparent places of stars, using the National machine. Checked copy, printed by the machine in precisely the form required by the printer, is produced in a time which compares favourably with other methods.

5. V. Vand, "A mechanical calculating machine for X-ray structure factors," *Nature*, v. 163, 29 Jan. 1949, p. 169–170. 17.4 × 25.5 cm.

First sentences: A large mechanical machine for calculation of X-ray crystallographic structure factors has now been completed in our laboratories [Lever Bros. and Unilever Ltd., Port Sunlight, Cheshire] and is running satisfactorily. The machine is of a tide-predictor type, and it deals with up to 24 harmonic components at a time. The expression calculated is $F(hkl) = \sum_i f_i \cos 2\pi(hx_i + ky_i + lz_i)$.

6. Heinz Wittke, *Die Rechenmaschine und ihre Rechentechnik. Eine Einführung und ein Übungsbuch mit ausgewählten Anwendungsbeispielen aus der Geodäsie, Geometrie und angewandten Mathematik. 2. Auflage. Mit 52 Abbildungen, 12 Formularen und 1 Tabellenanhang.* (*Sammlung Wichmann*, v. 12). Berlin, Herbert Wichmann, 1948, viii, 160 p. 17.4 × 24.4 cm. "Veröffentlicht unter Lizenz-Nr. 136 der Sowjetischen Militärverwaltung in Deutschland."

This is the so called second edition of which we listed the first edition in *MTAC*, v. 3, p. 390. The first edition was neatly bound in full boards. The second edition is a poor paper

bound offset print of the first edition, authorized by license of the Soviet military powers. The first 70 p. include a description of various desk calculating machines, of problems with which they may deal, and also lists of references to literature appearing before 1942.

R. C. A.

NOTES

106. GEORGE NEVILLE WATSON—TABLE MAKER.—This distinguished British Mathematician and Table Maker was born in Westward Ho, Devonshire, 31 January 1886. He was a student at Trinity College, Cambridge, senior wrangler, 1907; class I (div. ii), mathematical tripos, part II, 1908; Smith's Prizeman, 1909; fellow of Trinity, 1910–1916.

He was an assistant professor of pure mathematics at University College, London, 1915–18; professor of mathematics at the University of Birmingham since 1918, of pure mathematics since 1937; gold medallist of the Danish Royal Academy of Sciences, 1912; a fellow of the Royal Society of London, 1919; honorary secretary, 1919–33, and president, 1933–35, of the London Mathematical Society; president of the Mathematical Association, 1932–34; SYLVESTER medallist of the Royal Society, 1946; and DE MORGAN medallist of the London Mathematical Society, 1947. He is also a Sc.D. of Univ. of Cambridge, D.Sc. Univ. of London, M.Sc. Univ. of Birmingham, Hon. LL.D. Univ. of Edinburgh, Hon. Sc.D. Univ. of Dublin.

The following is a list of Professor Watson's tables, and of some papers with numerical results of some importance:

1. (with G. A. Schott) "Asymptotic formulae occurring in electron theory," *Quart. Jn. Math.*, v. 47, 1916, p. 311–333. Various numerical results are given.

2. "The sum of a series of cosecants," *Phil. Mag.*, s. 6, v. 31, 1916, p. 111–118. $S_n = \sum_{m=1}^{n-1} \csc(m\pi/n)$ is tabulated for $n = [2(1)30(5)100, 360, 1000; 5D]$.

3. [Tables connected with gamma functions], BAAS, *Report*, 1916, p. 123–126. Four 10D tables: (a) $10 + \ln \Gamma(1 + x)$, $x = .005(.005)1$; (b) $10 + \int_0^x \log \Gamma(1 + t)dt$, $x = .01(.01)1$; (c) $\psi(x) = d \ln \Gamma(x)/dx$, $x = 1(1)101$; $\psi(x)$ for $x = 1.5(1)100.5$.

4. "The zeros of Bessel functions," R. Soc. London, *Proc.*, v. 94A, 1918, p. 190–206. Tables of $J_{\pm\frac{1}{3}}(x)$, $U_{\frac{1}{3}}(x)$, $-V_{\frac{1}{3}}(x)$, $-V_{\frac{1}{3}}(x)/U_{\frac{1}{3}}(x)$, for $x = [0(.05)2(.2)8; 4D]$.

5. "Bessel functions of equal order and argument," *Phil. Mag.*, s. 6, v. 35, 1918, p. 364–370. Table of $n\int_0^1 J_n(nx)dx$, $n = [1(2)23; 7D]$.

6. *A Treatise on the Theory of Bessel Functions*, Cambridge, England, 1922; second ed., New York, 1944; reprinted Feb. 1945, Apr. 1948; tables, p. 665–752. For tabular errors in this v. see *MTAC*, v. 2, p. 49–51.

T. I: $J_0(x)$, $Y_0(x)$, $J_1(x)$ and $Y_1(x)$, for $x = [0(.02)16; 7D]$. The values of $J_0(x)$, $J_1(x)$ up to 15.5 were taken from MEISSEL's 12D table (1889) while the values of $Y_0(x)$ and $Y_1(x)$ were computed partly by interpolation in ALDIS' table of $G_0(x)$ and $G_1(x)$ (1900). T. I gives also, for the same range of argument, 7D values of $|H_n^{(1)}(x)|$, $\mathbf{H}_n(x)$, $n = 0, 1$, and of $\arg H_n^{(1)}(x)$, $n = 0, 1$ to the nearest 0".01.

T. II consists of tables of $e^{-x}I_0(x)$, $e^{-x}I_1(x)$, $e^xK_0(x)$, and $e^xK_1(x)$, e^x, for $x = [0(.02)16; 7D]$. The 8S or 9S table of e^x was constructed with the help of Newman's 12–18D table of e^{-x} (1883).

T. III consists of $J_{\frac{1}{3}}(x)$, $Y_{\frac{1}{3}}(x)$, $|H_{\frac{1}{3}}^{(1)}(x)|$, and $\arg H_{\frac{1}{3}}^{(1)}(x)$, of the same scope as T. I; a table of $e^xK_{\frac{1}{3}}(x)$ is also included.

T. IV gives 7D values of $J_n(x)$, for $n = 2(1)5$, $x = .1(.1)5$; 6D values of $J_n(x)$, $n = 0(1)20$, for $x = 1(1)12$; 7S at least, or 7D values of $Y_n(x)$ for $n = 0(1)10$, and $x = 0(.1)5$, and 7D for $n = 0(1)13$, $x = 6(1)12$; 7D values of $e^{-x}I_n(x)$ for $n = 2(1)5$, $x = .1(.1)5$; 7S at least, or 7D

values of $K_n(x)$ for $n = 0(1)10$, $x = 0(.1)5$. The 6D values of $J_n(x)$ are taken from LOMMEL's table (1885) with some corrections, and some values of $K_n(x)$ were taken from ISHERWOOD's table (1904).

T. V gives Lommel's 6D table (1886) of $J_n(x)$, $n = -6\frac{1}{2}(1) + 6\frac{1}{2}$, $x = 1(1)50$, and $n = 7\frac{1}{2}(1)18\frac{1}{2}$, $x = 1(1)20$, and FRESNEL's integrals, $x = .5(.5)50$, and $x = .02(.02)1$ with some modifications and corrections, and with the first 16 maxima and minima of the integrals. There are about 30 errors here.

T. VI gives the 7D values of $J_n(n)$, $-Y_n(n)$, $J_n'(n)$, $Y_n'(n)$ and $n^{\frac{1}{3}}J_n(n)$, $-n^{\frac{1}{3}}Y_n(n)$, $n^{\frac{1}{3}}J_n'(n)$, $n^{\frac{1}{3}}Y_n'(n)$ for $n = 1(1)50$.

T. VII gives the first 40 zeros of $J_n(x)$ and $Y_n(x)$ to 7D, for $n = 0(1)5$, $\frac{1}{2}$; and also of $J_{-\frac{1}{2}}(x) + J_{\frac{1}{2}}(x)$, and of $J_{-\frac{1}{2}}(x) - J_{\frac{1}{2}}(x)$.

T. VIII gives the 7D values of $\frac{1}{2}\pi\int_0^x J_0(t)dt$, $\frac{1}{2}\pi\int_0^x Y_0(t)dt$ for $x = .02(.02)1$, with the first 16 maxima and minima of the integrals, to 7D.

7. "The sum of series of cosecants," *Phil. Mag.*, s. 6, v. 45, 1923, p. 577–581. Compare no. 2. $S_n^{(3)} = \sum_{m=1}^{n-1} \csc^3(m\pi/n)$, table for $n = [2(1)30(5)100, 360, 1000; 5D]$.

8. "Theorems stated by Ramanujan (V): approximations connected with e^x," London Math. Soc., *Proc.*, s. 2, v. 29, 1929, p. 293–308. Solutions of $ue^{1-u} = Ue^{1-U} = e^{-t}$ with values of $\phi(t) = \frac{135}{8}\left\{\frac{U}{(U-1)^3} - \frac{u}{(1-u)^3}\right\}$, tables of u, U, $\phi(t)$, $-t^{-1}\ln\phi(t)$, for $t = [0(.02)-1(.1)11; 7D]$.

9. "The constants of Landau and Lebesgue," *Quart. Jn. Math.*, Oxford s., v. 1, 1930, p. 310–318. Contains certain numerical results.

10. "The use of series of Bessel functions in problems connected with cylindrical wind-tunnels," R. Soc. London, *Proc.*, v. 130A, 1930, p. 29–37. A few numerical results.

11. "Du Bois-Reymond's constants," *Quart. Jn. Math.*, Oxford s., v. 4, 1933, p. 140–146. Various computed results are given.

12. "Two tables of partitions," London Math. Soc., *Proc.*, s. 2, v. 42, 1937, p. 550–556. Tables of the number of partitions of n into odd parts and into distinct odd parts for $n = 0(1)400$.

13. "Tabellazione di una particolare funzione definita da un integrale improprio," R. Accad. naz. Lincei, *Cl. d. sci. fis., matem. e nat., Rendiconti*, s. 6, v. 27, 1938, p. 525–528. Table of $\phi(\lambda) = \int_0^\infty [x + \frac{1}{2} - (x^2 + x)^{\frac{1}{2}}] \sin \lambda x dx$, $\lambda = [0(.01).4(.1)32; 7D]$.

14. "Three triple integrals," *Quart. Jn. Math.*, Oxford s., v. 10, 1939, p. 266–276. Certain numerical results.

15. "A table of Ramanujan's function $\tau(n)$," London Math. Soc., *Proc.*, s. 2, v. 51, 1949, p. 1–13. See *MTAC*, v. 3, p. 468. Gives a table of $\tau(n)$ for $n = 1(1)1000$; also $\tau(n)n^{-11/2}$ to 5D.

We are indeed grateful to Professor BROADBENT, the editor of the *Mathematical Gazette*, for consenting to allow us to reproduce the portrait of Professor Watson published as a frontispiece to the *Gazette*, v. 18, 1934.

THE EDITORS

107. THE LEMNISCATE CONSTANT.—By this constant $\tilde{\omega}$ is usually meant twice the number

$$\tfrac{1}{2}\tilde{\omega} = A = \tfrac{1}{4}(2\pi)^{-\frac{1}{2}}\{\Gamma(\tfrac{1}{4})\}^2 = \tfrac{1}{4}B(\tfrac{1}{4}, \tfrac{1}{2}) = F(\tfrac{1}{4}, -\tfrac{1}{2}; \tfrac{5}{4}; 1)$$
$$= \left\{\binom{-\frac{1}{4}}{\frac{1}{4}}\right\}^{-1} = \tfrac{1}{2}\pi\vartheta_4^2(0|i) = \Pi(\tfrac{1}{4})\Pi(-\tfrac{1}{2})/\Pi(-\tfrac{1}{4}) = \int_0^1 (1 - x^4)^{-\frac{1}{2}}\,dx.$$

It derives its name from the fact that it plays the rôle of π in the rectification of the lemniscate. In fact the length of the lemniscate of diameter D is

$\tilde{\omega}D$. This constant occurs in a good many connections in the theory of elliptic functions and in number theory. The values

$$(1)\ A = 1.311031,$$
$$(2)\ A = 1.31102\ 87771\ 46059\ 87,$$
$$(3)\ A = 1.31102\ 87771\ 46059\ 90680\ 3207,$$

were given by EULER, STIRLING[1] and GAUSS[2] respectively. The value (3) is given in the FMR *Index*. WRENCH [*MTAC*, v. 3, p. 202] has pointed out that this value is incorrect beyond the 17th decimal.

In order to verify certain identities involving this constant with a number of numerical functions the writer has made two independent redeterminations of A using the two lacunary series

$$A = \tfrac{1}{2}\pi\{\sum_{n=-\infty}^{\infty}(-1)^n e^{-\pi n^2}\}^2 = 2^{\frac{1}{2}}\pi e^{-\frac{1}{4}\pi}\{\sum_{n=-\infty}^{\infty}(-1)^n e^{-2\pi(2n^2+n)}\}^2$$

the first of which was used by Gauss. These calculations agree to 52D and give

$$A = 1.31102\ 87771\ 46059\ 90523\ 24197$$
$$94945\ 55970\ 68413\ 77475\ 71581.$$

Gauss gave also the constant

$$B = \tfrac{1}{2}\pi/\tilde{\omega} = \tfrac{1}{4}\pi/A = (2\pi)^{-\frac{1}{2}}\{\Gamma(\tfrac{3}{4})\}^2 = \int_0^1 x^2(1-x^4)^{-\frac{1}{2}}dx$$

correctly to 20D. More accurately we have

$$B = .59907\ 01173\ 67796\ 10371\ 99612$$
$$46140\ 16193\ 91136\ 06331\ 60783.$$

As a byproduct of these calculations we obtain easily the following values of $\Gamma(\frac{1}{4})$ and $\Gamma(\frac{3}{4})$:

$$\Gamma(\tfrac{1}{4}) = 3.62560\ 99082\ 21908\ 31193\ 06851$$
$$55867\ 67200\ 29951\ 67682\ 88007$$
$$\Gamma(\tfrac{3}{4}) = 1.22541\ 67024\ 65177\ 64512\ 90983$$
$$03362\ 89052\ 68512\ 39248\ 10807.$$

D. H. L.

[1] J. STIRLING, *Methodus Differentialis*, London, 1730, p. 58; English transl. by F. HOLLIDAY, London, 1749, p. 51.
[2] K. F. GAUSS, *Werke*, v. 3, Göttingen, 1866, p. 413.

108. NEWMAN AND GEORGE ELIOT.—We have made various references to the table-maker, FRANCIS WILLIAM NEWMAN (1805–1897), brother of the Cardinal; see for example *MTAC*, v. 1, p. 454–459, v. 3, p. 201, 257, 451. At the first of these references we noted (p. 455) that he was professor of Latin at University College, London, 1846–1869. In the *Times Literary Supplement*, London, v. 48, April 30, 1949, p. 281, there is an interesting letter by KATHLEEN TILLOTSON[1] noting, what had escaped her biographers, that GEORGE ELIOT (1819–1880) was one of the earliest students of the Ladies' College at Bedford Square, London, which opened in October, 1849 and is now Bedford College for Women. During 1851 at least, writes Mrs. Tillotson, she took courses there in History, German, French, Elocution, and Mathe-

matics. Her mathematics, and possibly her history, was with Newman, who taught mathematics at the College January–June 1851. In a letter of March 27, 1874 George Eliot remembered her interest in Francis Newman's books "in far-off days . . . and . . . the awe I had of him as a lecturer in Mathematics at the 'Ladies' College.' " Mrs. Tillotson continues: "He had lectured in Ancient History since the opening of the college, and it is natural to suppose that it was partly his name . . . that drew MARY ANN EVANS to the Ladies' College."

In the *Times Literary Supplement*, June 3, 1949, p. 365, Prof. GORDON S. HAIGHT of Yale University quotes from a letter "which has just come to light" written by George Eliot Jan. 28, 1851. A passage quoted is as follows: "I am attending Professor Newman's course of lectures on Geometry at the Ladies' College every Monday and Thursday. You will say that I can't afford this, which is 'dreadfully true'—but the fact is that I happened to say I should like to do so and good-natured Mr. CHAPMAN[2] went straightway and bought me a ticket which he begged me to accept. I refused to accept it—and have paid for it—wherefore I must stint myself in some direction—clearly in white gloves and probably in clean collars."

R. C. A.

[1] Mrs. TILLOTSON is a coeditor of the *Works of Michael Drayton*, and wife of GEOFFREY TILLOTSON, professor of English Literature in the University of London.

[2] JOHN CHAPMAN, the publisher, at whose home George Eliot resided Jan. 8 to March 24, 1851.

109. RHETICUS, WITH SPECIAL REFERENCE TO HIS OPUS PALATINUM.—Before the twentieth century Rheticus was the greatest calculator of mathematical tables who ever lived. He was born just a century before Napier published his work on logarithms in 1614, and his tables were posthumously published in 1596 and 1613.

Since the ordinary inquirer would in more than one direction find considerable difficulty in deriving a fairly complete outline of the life and work of Rheticus, such an outline is here presented, together with a somewhat extended list of sources (everything has been personally inspected) from which fuller information may be derived. In this outline some errors in these sources have been corrected, a few new facts delineated, and our previous sketch of PITISCUS (*MTAC*, v. 3, p. 390–397) amplified.

Rheticus was one of many mathematicians named after the places of their birth. GEORG JOACHIM was born 16 Feb. 1514 at Feldkirchen in Vorarlberg, in the westernmost part of Austria-Hungary, close to the Swiss border and formerly in the Roman province of Rhaetia. Georg's mother seems to have been connected with an Italian noble family of means, named PORRI,[25] and thus a trip to Italy with his parents as a child was what might be expected, and suggested a family in comfortable circumstances. Joachim's zest for advanced education seems to have been stimulated by studies at Zürich, Switzerland. Hence in 1532 the fame of the comparatively new University of Wittenberg, where the great reformer PHILIP MELANCHTHON (1497–1560) was professor of Greek, and MARTIN LUTHER (1483–1546) was professor of Theology, drew the eighteen-year old youth to register as a student. In April 1536 he received the equivalent of a doctor of philosophy degree, the official records stating that he was "mathematicus excellens."

It was exactly at this time that he first added the surname Rheticus. (It may be noted that in an outstanding German reference work[16] there are two biographies of Rheticus, by different authors, one under "Joachim" and the other under "Rheticus.")

In May 1536 Rheticus went to Nuremberg to study with JOHANN SCHÖNER (1477–1547) professor of mathematics at the gymnasium there, and a former pupil of REGIOMONTANUS and WERNER.[22] Later he went to Tübingen where he received the call back to his University.[14] The professor of mathematics at Wittenberg having died in 1536 Melanchthon, then being Dean of the faculty, appointed for that autumn two new professors: one ERASMUS REINHOLD (1511–1553) for the higher mathematics (including astronomy), and the other, Dr. Rheticus, then 22 years of age, for elementary mathematics. After teaching there with some repute for about three years Rheticus received leave of absence in May 1539 and went to Frauenburg, Prussia, as an assistant to COPERNICUS (1473–1543), whose doctrines he soon advocated with much zeal and personal risk.

Copernicus was very reluctant to publish his epoch-making astronomical work, *De revolutionibus orbium caelestium*, but publicity achieved for its contents by Rheticus led him to change his mind and to permit publication. This publicity was a survey of the principal features of the new astronomy, cast in the shape of a letter to his former teacher SCHÖNER at Nuremberg. This was published anonymously at Dantzig in 1540, as a *Narratio prima*, and reprinted at Basel in 1541 with the name of Rheticus as author. Of this *Narratio* there were nine later editions, the eleventh being that of ROSEN in English.[28] Rheticus left Frauenburg at the end of September 1541, resumed his teaching in Wittenberg, and served as dean of the arts faculty there in the early months of 1542.[25]

Chapters 12, 13, 14 of the first of the six books of the *De revolutionibus* of Copernicus were devoted to the trigonometrical material, useful for the later discussion of astronomical problems. In the 12th chapter was a 5D table, with differences, of the sines of all angles in the quadrant at interval 10′. In 1542 Rheticus had chapters 12 and 13 printed at Wittenberg with the table replaced by his own 7D table of sines and cosines for every minute in the quadrant.

Rheticus left Wittenberg in 1542 and went to Nuremberg where the great work of Copernicus was being printed; the early part was set up under his direction. Rheticus placed the first completed copy of the work in the hands of Copernicus a few hours before he died in May 1543. The second edition of *De revolutionibus*, edited by Rheticus appeared in 1566, with the appended third edition of his *Narratio*.

In securing the publication of *De revolutionibus* Rheticus bears to Copernicus a relation similar to that of HALLEY to NEWTON, in the following century, for assuring the publication of the *Principia*.

The following statement of Müller[23] has been frequently quoted: "Without Rheticus we would know no Copernicus, without Copernicus no Kepler, without Kepler no Newton!"

At Nuremberg in 1542 Rheticus also published *Orationes de astronomia, geographia et physica*. He taught at the University of Leipzig from 1542 to 1551. In this latter year was published his *Canon doctrinae triangulorum*.[10, 19]

Leipzig, 24 p., to which we have earlier referred,[30] p. 394. In this quarto-format publication of 12 leaves, not listed in any of the bibliographies, p. 4–17 are occupied with a 7D Canon for all six trigonometric functions, at interval 10′, and this is followed by a dialogue between Philomathes, a supposed friend of Rheticus, and Hospes, his pupil. This was the first table (a) in which all trigonometric functions (or even sin, tan, sec) were brought together, or (b) in which the semiquadrantal arrangement was used. Both this canon and the undated reprint at Basle (apparently in 1580), are in the British Museum. With Rheticus our names for the trigonometric functions were not used; sines and cosines were perpendiculars and bases to a hypotenuse 10 000 000; the secants and tangents were hypotenuses and perpendiculars to a base 10 000 000; and the cosecants and cotangents were hypotenuses and bases to a perpendicular 10 000 000. Rheticus was the first to define his functions by means of the right-angled triangle without any reference to the circle.

In 1551 Rheticus went to Prague; and in 1554 he lectured at the University of Vienna,[24] but in 1557 he settled in Cracow, where he spent most of the rest of his life. He died during a visit to Cassau, in Hungary, on 4 Dec. 1576 in the sixty third year of his age.[31] The great scarcity of the works of Rheticus (the second edition of *De revolutionibus* is much rarer than the first) is due to the fact that in 1550 all of his works were put on the *Index Expurgatorius*.

During the last 25 years of his life Rheticus devoted himself principally to the calculation of monumental trigonometric tables. Contributions of financial assistance from MAXIMILIAN II of Austria (who died in 1576) and several Hungarian nobles enabled Rheticus to procure a number of computers for 12 years (3 and WOLF[15]).

The Opus Palatinum. During this period of intense activity Rheticus had long known of a young mathematician, a native of Magdeburg, LUCIUS VALENTIN OTHO (1550?–1605?) well known[18] to PRÄTORIUS at the University of Wittenberg. Towards 1575 Rheticus gladly accepted the services as assistant which Otho offered. And thus it turned out that when Rheticus died in December 1576 Otho became heir to all of the scientific mss. of Rheticus. Rheticus has told us that when Otho came to him he was the same age (25) as when he (Rheticus) became an assistant to Copernicus ("Profecto in eadem aetate ad me venis, qua ego ad Copernicum veni"). In this way we learn that Otho was born about 1550. He became a professor of mathematics (following Prätorius) at the University of Wittenberg, and was continually seeking to procure the means for publishing the great tables which were practically complete. This was finally arranged with FREDERICK IV,[31] Elector Palatine, to whom the work is dedicated. Thus 20 years after the death of Rheticus, in 1596, his *Opvs Palatinvm de triangvlis a Georgio Ioachimo Rhetico coeptvm: L. Valentinvs Otho, Principis Palatini Friderici IV. Electoris Mathematicvs consvmmavit.* was published at Neustadt, now in Bavaria, in about 1446 folio pages, 22.4 × 36.7 cm. It seems to have been originally bound in vellum in two volumes. Five of the seven parts were in the first volume.

After the title-page (which we here reproduce) and 9 preliminary leaves of dedication and preface by Otho are 7 main sections each (after the

Title-Page of RHETICUS, *Opus Palatinum*, 1596.

second) separately paged, and for each of the first 5 sections there is at least one separately dated title-page. The first three of these sections, by Rheticus, are as follows:

I. *De Fabrica canonis doctrinae triangvlorvm.* The title page, with verso blank, is followed by p. 3–86. P. 45 is printed and folded so as to occupy the verso of a blank leaf and the recto of p. 46, making an extra leaf. Pages 34, 35, 37 and 38 are incorrectly numbered, while the verso of p. 85 also bears the number 85 instead of the correct number 86. Here are three books on the construction of the canon.

II. *Triqvetrvm rectarvm linearvm in planitie cvm angvlo recto,* p. 86–104. One book on plane triangles.

III. *De triangvlis globi cvm angvlo recto.* 146 p. Four books on right-angled spherical triangles. The verso of p. 21 is 23, the recto of p. 23 is 24, and the verso of p. 24 is blank. P. 57 is numbered 60. P. 72–88 are numbered 81, 83, 82, 85–98. Then p. 89–106 are respectively numbered: 100–103, 95, 105, 106, 98, 108–115, 101, 102. Here a blank leaf is inserted which we shall call p. 107–108. Then corresponding to p. 109–146 are pages numbered 103–140. In addition there is inserted between p. 115(109)–116(110) a printed half-page containing matter left out of the text in the printing.

Following this material by Rheticus are 5 books by Otho on oblique spherical triangles and 3 subsidiary astronomical tables which Otho calls *meteoroscopia.*

IV. *De triangvlis globi sine angvlo recto libri qvinqve.* The title page with verso blank is followed by p. 1–264, then commencing on the next recto the pages run from 264 (repeated) to 341 on a verso. The recto of the next leaf contains the following: "Neostadii in Palatinatv. Excudebat Matthaeus Harnisius [printer's device]. Anno Salutis MD XCVI." The verso of the last mentioned leaf is blank and it is followed by a blank leaf.

V. *Meteoroscopivm numerorvm primvm.* p. 1–91. *Meteoroscopivm numerorvm secvndvm.* p. 92–105. *Meteoroscopivm numerorvm tertivm.* p. 106–121. Each of these three parts has its own dated title page. Then follows a closely packed recto page entitled "Errata sic corrige," corrections to be made in this volume I.

Volume II is devoted to tables, each with an undated title page. The first of these is the great Rheticus 10D tables at interval 10″, for all six of the trigonometric functions.

VI. *Georgii Ioachimi Rhaetici Magnus canon doctrinae triangvlorvm ad decades secvndorvm et ad partes* j0000000000. All of this is printed in red, after the name of Rheticus in black, 554 p. The table occupies p. 2–541. Then come Errata p. 542–554, incorrectly numbered 142–147, 147 repeated, 549–554. P. 210 reads 201, 290 reads 390, 309 reads 314, 398 reads 498, 512 reads 125.

Following these errata is a 7D table at interval 10″, of cotangents and cosecants for the first half of the quadrant:

VII. *Tertia series magni canonis doctrinae triangvlorvm in qvo triqvetri cvm angvlo recto in planitie minvs latvs inclvdentivm angvlvm rectum ponitur partium* 10000000. After the first two words in black, the rest of the title is red. 181 p. P. 83 is incorrectly numbered 27.

Concerning this section VII DeMorgan remarks[11] that its insertion displayed "merely the editor's want of judgment; it is clearly nothing but a previous attempt made before the larger plan was resolved on, and is much less accurate than the great table to ten places."

DELAMBRE,[8] p. 2, states that in III–IV Rheticus and his disciple are the most prolix and obscure authors he had ever met; and that their 500 folio pages could be reduced to 10.

Pitiscus Revision of the Opus Palatinum Canon. Shortly after the *Opus Palatinum* was published it was found that in VI the tangents and secants near the end of the quadrant were very inaccurate. Possibly through representations of PITISCUS, then chaplain and former teacher of Frederick

IV, the Elector Palatine put Pitiscus in charge of the revision of this part of the canon. From the account which Rheticus gave of the construction of the canon in I, especially book 3, it is clear that the computation of the tangents and secants were based on a table of sines and cosines, at interval 10″, and extent >10D. Hence Pitiscus, perhaps about 1602 (when he was 41 years old), sought the ms. of Otho "then an old man" (say La Lande, Delambre,[8] p. 17, DeMorgan,[11] Glaisher[13]—based on the statement of Pitiscus: "ob memoriae senilis debilitatem"—who could not remember where the ms. was). He thought that perhaps he had left it at Wittenberg, and accordingly Pitiscus sent a messenger there to search for it; but after considerable expense had been incurred he returned without it. In 1602 Otho was about 52 years old, just 11 years older than Pitiscus. While the condition of his health may have affected his memory it is certainly improper[31] to refer to Otho at this time as an "old man."

After the death of Otho, at Heidelberg, about 1605, when the mss. of Rheticus which had been in his possession passed into the hands of the orientalist and astronomer at Heidelberg, JACOB CHRISTMANN, he found that they included not only the sine canon which had been considered lost, but also the original manuscript of the *De revolutionibus* of Copernicus, now preserved in Prague.

As soon as Pitiscus learned of this canon discovery he examined the mss. page for page, although they were in very bad condition, and to his great satisfaction he found

(1) the ten-second canon of sines, 15D, Δ^3;
(2) sines for every second of the first and last degrees of the quadrant, 15D, Δ^2;
(3) the commencement of a canon for every ten seconds of tangents and secants, 15D, Δ^2;
(4) a complete minute-canon of sines, tangents, and secants, 15D.

From this list of tables, taken in conjunction with VI and VII, we are enabled to view as a whole the colossal sixteenth century computations which Rheticus achieved, and which must later have been of extraordinary value to the astronomer.

With the canon (1) in hand Pitiscus recomputed to 11D all of the tangents and secants of VI in the defective region from 83° to the end of the quadrant. Then 86 pages were reprinted: p. 1, the new title-page, and 2–86, the revised table. For the new title-page to that already given in VI was added, 1.

Recens emendatus à Bartholomaeo Pitisco Silesio. Addita est brevis commonefactio de fabrica & vsu huius Canonis. Quae est summa doctrinae, & quasi nucleus, totius operis Palatini. Canon hic, unà cum brevi commonefactione de eius fabrica & vsu etiam separatim ab opere Palatino venditur. In Bibliopoleio Harnischiano.

This is a sort of fly-title, undated; but the accompanying 19-page publication has the following title page, 2: *Bartholomaei Pitisci Grünbergensis Silesii Brevis et Perspicua Commonefactio de Fabrica et vsv magni canonis doctrinae Triangulorum Georgii Ioachimi Rhetici.* [Printer's decoration]. *Neostadii, Typis Nicolai Schrammii.* MDCVII.

The 86 pages of 1 and the 19 pages of 2 were issued, as stated above, as a separate publication, but also with copies of the complete work with

the original pages 1–86 in VI eliminated and replaced by 1, and with 2 following the revised VI, or VII.[32] It is the latter arrangement which is found in the only three existing copies of 1–2, which I have been able to trace, and to which I shall presently refer.

Otho's Death. Since on page 5 of 2 there seems to be definite reference to the Rheticus sine canon being in the hands of Christmann, this suggests that Otho had died before 1607 and hence I suggested that he may have died about 1605 and that Pitiscus may have questioned him concerning the table about 1602. While I arrived at this conclusion as to a possible date of Otho's death independently I was later interested to find that TROPFKE had reached the same result in the second ed. of his *Geschichte der Elementar-Mathematik*, v. 4, 1923. The only definite older reference to the death of Otho is in the "Ad lectorem" of the work we shall now consider, namely:

3. The Rheticus-Pitiscus *Thesaurus Mathematicus*. After his discovery of the new Rheticus tables Pitiscus started to prepare a second work which was finally published in 1613 and contained the following four parts.

A. (Rheticus) Canon of sines for every 10″, 15D, Δ^3, p. 2–271;

B. (Rheticus) Sines for 0(1″)1°, 89°(1″)90°, 15D, Δ^3, p. 2–61;

C. (Pitiscus) The fundamental series from which the rest were calculated to 22D, p. 1–10;

D. (Pitiscus) The sines to 22D, Δ^4 or Δ^5, for every tenth, thirtieth and fiftieth second in the first 35 minutes, p. 11–15.

Each of the parts *C*, *D* has its own correctly dated title-page 1613; but the title-page for *B* as well as that for the whole volume is given incorrectly as MDXIII. The volume title-page, in black and red, begins *Thesaurus Mathematicus sive Canon Sinuum ad radium* 1.00000.00000.00000. . . . Frankfurt. The volume is a large one 24×35 cm. It appeared shortly before the death of Pitiscus 2 July 1613 in his fifty second year. From what has been presented above the great contribution which Pitiscus made in uncovering, and presenting work of Rheticus in meticulously correct form, was indeed a notable contribution to science.

More information concerning this volume is given in *MTAC*,[30] p. 395–396.

One had to wait for nearly three and a half centuries before the tables of Rheticus were finally superseded by those of ANDOYER (1915–1918).

Known copies of the 1607 Rheticus-Pitiscus work.

The three known copies of this work are in Edinburgh, at the Crawford Library of the Royal Observatory, which we shall call the ECO copy; in Washington, at the Naval Observatory, the WNO copy; and in the private library of Mr. WILLIAM D. MORGAN of St. Paul, Minnesota—the StPM copy.

For the preparation of this article I have been deeply indebted to Mr. Morgan for allowing me to have his StPM copy for leisurely study. In at least one respect it is the finest copy in a single library. It consists of three volumes, purchased at different times from three different firms: (i) almost certainly the volume of I–V, bound in contemporary white vellum with

leather tiers as originally issued in 1596, 36.7 cm. tall, and thus indicating that at least some copies of the original were bound in two v. and divided in that way; (ii) containing V–VII, as published in 1596, 1.7 cm. less tall than (i); (iii) containing 1 along with the remaining 454 pages of the trigonometric canon, 2, and 3. This third v. is 3 cm. less tall than (i), and at least 1.3 cm. less tall than 3 when it was originally published. In StPM the many fundamental changes in VI of Rheticus which Pitiscus made in 1 may be readily observed. In this respect and in the superb condition of (i) StPM is outstanding.

As noted above, in both ECO and WNO there is replacement of the first 86 pages of VI by 1. The great kindness of Mrs. GRACE O. SAVAGE, the librarian at the Naval Observatory, in responding to my numerous queries concerning WNO, has been much appreciated. This copy, as tall as (i) and with wider pages, is also bound in vellum but with different ordering of material. The Library of Congress Rheticus lacking the Pitiscus revision, 36.5 cm. tall, is practically as tall as (i). Brown University has film copies of the Library of Congress Rheticus, and of Mr. Morgan's 1607 Pitiscus items.

We have earlier (*MTAC*, v. 3, p. 396) indicated the reason for the scholar's great interest in the ECO copy.

R. C. A.

[1] ADRIAAN VAN ROOMEN (or ADRIANUS ROMANUS), *Ideae mathematicae pars prima, sive methodus polygonorum*. Louvain and Antwerp, 1593. In preliminary material Roomen has (i) "lectori philomathi" in which he has words of praise for 15 of the principal mathematicians of his time (including Rheticus and VALENTIN OTHO), and (ii) a letter written by Rheticus in 1568 to PETRUS RAMUS (1515–1572), telling not only of his great tabular calculations but also of such works as the following which he planned to publish: (a) Observation of Phenomena; (b) German astronomy; (c) Natural Philosophy; (d) Foundations of Chemistry. See H. BOSMANS, "Romain (Adrien)," *Biogr. Nationale . . . de Belgique*. Brussels, v. 19, 1907, col. 855; also A. MÜLLER,[12] p. 40.

[2] J. J. C. F. DE LALANDE, "Lettre sur tables de sinus extrêmement rares," *Jn. des Sçavans*, Paris, Sept., 1771, p. 579–584. The Rheticus-Pitiscus *Thesavrvs Mathematicvs*, 1613.

[3] A. G. KÄSTNER, *Geschichte der Mathematik*. Göttingen, v. 1, 1796, p. 561f, 590f; v. 2, 1797, p. 368.

[4] C. HUTTON, *Mathematical Tables: containing Common, Hyperbolic, and Logistic Logarithms. . . . To which is prefixed a large and original history of the discoveries and writings relating to those subjects. . . .* London, 1785, p. 9–11; also in his *Tracts on Mathematical and Philosophical Subjects*. V. 1, London, 1812, p. 290–293.

[5] JEAN BERNOULLI, "Analyse de l'Opus palatinum de Rheticus & du Thesaurus mathematicus de Pitiscus," Akad. d. Wissen., Berlin, *Nouveaux Mémoires*, 1786, p. 10–33.

[6] J. B. DELAMBRE, "Rapport sur les grandes tables trigonométriques décimales du cadastre," Acad. d. Sci., Paris, *Mémoires, Sci. Math. et Phys.*, v. 5, 1804, p. 56–66.

[7] G. C. F. RICHE DE PRONY, "Éclaircissemens sur un point de l'histoire des tables trigonométriques," Acad. d. Sci., *Mémoires, Sci. Math. et Phys.*, v. 5, 1804, p. 67–93.

[8] J. B. DELAMBRE, *Histoire de l'Astronomie Moderne*, v. 2, Paris, 1820, p. 1–127: "Grandes tables trigonométriques en nombres naturels—Rheticus."

[9] A. DEMORGAN, "Rheticus," *Penny Cycl.*, v. 19, 1841, also *The English Cycl., Biography*, v. 5, London, 1857.

[10] A. DEMORGAN, "On the almost total disappearance of the earliest trigonometrical canon," RAS, *Mo. Not.*, v. 6, 1845, p. 221–228; reprinted with an addition in *Phil. Mag.*, s. 3, v. 26, 1845, p. 517–526.

[11] A. DEMORGAN, (i) "Table" in *Supplement to the Penny Cycl.*, v. 2, London, 1846, p. 596–597, 599–600. (ii) "Table," *English Cycl., Arts and Sci. Sect.*, v. 7, 1861, cols. 984, 988–989.

[12] J. C. POGGENDORFF, *Biogr.-Literar. Handwörterbuch*, v. 2, Leipzig, 1863.

[13] J. W. L. GLAISHER, *Report of the Committee on Mathematical Tables*, London, 1873, p. 43–45.

[14] F. HIPLER, "Die chorographie des Joachim Rheticus. Aus dem Autographon des Verfassers mit einer Einleitung herausgegeben," *Z. Math. Phys., Hist.-literar. Abth.*, v. 21, 1876, p. 125–150.

[16] R. WOLF, *Geschichte der Astronomie.* Munich, 1877, p. 204f, 236f, 242f, 296, 343f.

[16] *Allgemeine deutsche Biographie*, Leipzig, v. 14, 1881, p. 93–94: "Joachim, G." by K. C. BRUHNS; v. 28, 1889, p. 388–390; "Rheticus, G. J." by S. GÜNTHER.

[17] LEOPOLD PROWE, *Nicolaus Coppernicus.* Berlin, 2 v., 1883–1884; v. 1, Das Leben, v. 2, Urkunden. "For the biography of Copernicus and the social history of his times the book is extremely valuable, but Prowe's judgment in scientific matters was unreliable" (E. ROSEN).

[18] M. CURTZE, 1. "Zur Biographie des Rheticus," *Altpreussische Monatsschrift*, v. 31, 1894, p. 491–496. This article contains an extract from a Munich library ms., Codex latinus Monacensis no. 24101, written by JOHANNES PRÄTORIUS (1537–1616), inventor of the plane table used in surveying, who was professor of mathematics at the University of Wittenberg 1571–1576, and professor at the University of Altdorf from 1576 until his death. In this ms. we are told that in Aug. 1573 Otho, then mathematician for the landgrave of Hesse, came to his patron Prätorius with two notable approximations for π:

(a) $3.1415926537 > \pi > 3.14159265365$, published by VIETA 20 years later, in 1593;
(b) 355/113, which is correct to 6D.

The latter he had derived from the approximations 377/120, found by PTOLEMY, and 22/7 given by ARCHIMEDES, by subtractions of numerators and denominators. This is the earliest sure date for discovery of this approximation. This statement is made with full knowledge of the attribution of this result to: (a) the fifth Century Chinese astronomer TSU CH'UNG-CHIH by Y. MIKAMI, *The Development of Mathematics in China and Japan* (*Abh. z. Gesch. d. math. Wissen.*, Heft 30). Leipzig, 1913, p. 50; and (b) ADRIAEN ANTHONISZ (1527–1607) about 1583 by his son ADRIAEN METIUS in his *Arithmeticae libri duo et geometriae VI*, 1626, p. 51.

2. "Die abgekürzte Multiplication," *Z. Math. Phys., hist.-literar. Abt.*, v. 40, 1895, p. 7–13. Abridged multiplication and mss. of RHETICUS are here discussed. The names of OTHO, PITISCUS and WERNER, among others, also come up.

[19] K. HUNRATH, "Des Rheticus Canon doctrinae triangulorum und Vieta's canon mathematicus," *Abh. z. Gesch. d. Math.*, v. 9, 1899, p. 211–240.

[20] M. CANTOR, *Vorlesungen über Geschichte der Mathematik.* V. 2, second ed., Leipzig, 1900, p. 472–475, 600–603, etc.

[21] A. VON BRAUNMÜHL, *Vorlesungen über Geschichte der Trigonometrie.* Leipzig, v. 1, 1901, p. 140–149, 158–160, 212–226. (Das *Opus Palatinum* and Pitiscus), etc.; v. 2, 1903, 7 index references.

[22] JOHANN WERNER (1468–1528) of Nuremberg, astronomer and meteorologist, *De triangulis sphaericis libri quatuor.* Edited by A. A. BJÖRNBO; *De Meteoroscopiis libri sex, cum prooemio G. J. Rhetici.* Cracow 1557. Edited by J. WÜRSCHMIDT. *Abh. z. Gesch. d. Math. Wissen.*, v. 24, 1907, 1913.

[23] ADOLF MÜLLER, "Der Astronom und Mathematiker Georg Joachim Rheticus," *Vierteljahrsschrift für Geschichte und Landeskunde Vorarlbergs*, n. s., v. 2, 1918, p. 5–46. Valuable collection and study of material about the life of Rheticus.

[24] "Vorarlberger an in- und ausländischen Hochschulen vom Ausgange des XIII. bis zur Mitte des XVII. Jahrhunderts," *Forschungen zur Geschichte Vorarlbergs und Liechtensteins*, v. 1, 1920, p. 58, 128–130.

[25] FRANZ HAEFELE, "Zur Frage der Herkunft des Astronomen Georg Joachim de Porris," *Schriften des Vereines für Geschichte des Bodensees und seiner Umgebung*, Heft 55, Friedrichshafen a.B., 1927, p. 122–137. Some new material.

[26] MARTIN BILGERI, *Das Vorarlberger Schrifttum, und der Anteil des Landes am deutschen Geistesleben.* Vienna and Leipzig, 1936. "Georg Joachim, genannt Rheticus, der grosse Astronom und Mathematiker," p. 64–70. Nothing original.

[27] JOHANNES KEPLER, *Gesammelte Werke*, herausgegeben von MAX CASPAR, v. 1, Munich, 1938; Rheticus, *Narratio prima*, tenth edition, p. 88–131.

[28] EDWARD ROSEN, *Three Copernican Treatises: The Commentariolus of Copernicus, The Letter against Werner, The Narratio prima of Rheticus, translated with Introduction and Notes.* New York, Columbia Univ. Press, 1939. Admirable work. Introduction: "Georg Joachim Rheticus," p. 4–6; "The *Narratio Prima*," p. 9–11. Text of "*Narratio Prima*," p. 107–196, eleventh ed.

[29] ERNST ZINNER, *Entstehung und Ausbreitung der Coppernicanischen Lehre. Zum 200-jährigen Jubiläum der Friedrich-Alexander-Universität zu Erlangen.* Phys.-medizin. Soz. z. Erlangen, *Sitz.*, v. 74, 1943. Very many references to Rheticus in the index.

[30] R. C. ARCHIBALD, "Bartholomäus Pitiscus (1561–1613)," *MTAC*, v. 3, p. 390–397, 498–499, 1949.

[31] It is rare indeed that DeMorgan anywhere ever deviates in the slightest degree from exactness of statement; yet in connection with Rheticus three cases of this kind are to be noted. In no. 10 p. 228 it is stated that Rheticus died in the "sixty-first year of his age" instead of sixty-third. In nos. 9 (1857) and 11 it is stated that the Opus Palatinum was published "at the expense of the Emperor Maximilian," who had died 20 years earlier. About

1600 when Otho was about 50 years old DeMorgan refers to him in no. 11 as "then an old man."

[2] As DEMORGAN pointed out, the 86 pages of 1 are easily distinguishable by the inferiority of paper and type; the same is true of 2; also the corrected copies may be distinguished from the uncorrected ones in a moment as follows: look at the bottom of page 7, at the running titles of the columns. The uncorrected copy will have as it ought to have Basis Differentia Hypothenusa. But the *corrected* copy will have, as it ought not to have, Hypothenusa Differentia Basis.

QUERY

32. FRENCH AND RUSSIAN TRANSLATIONS OF A VEGA-BREMIKER TABLE. In RMT **635**, reference was made to the Carl Bremiker edition of Vega's *Logarithmisch-Trigonometrisches Handbuch*, first published in 1856. It was also noted that in 1857 an English translation of this edition by W. L. F. FISCHER, and an Italian translation by LUIGI CREMONA, were published. Who were the authors of the French translation of 1857 and of the Russian translation of 1858 also mentioned? The first two translations, as well as the 1857 (second Bremiker) German edition are in the Library of Brown University. In what library may the French or Russian translations be found?

R. C. A.

QUERIES—REPLIES

41. GIRARD AND SNELL TABLES (Q30, v. 3, p. 451).—ALBERT GIRARD, *Tables des Sinus, Tangentes, & Secantes, selon le Raid de 100 000 Parties. Avec un traicté succinct de la Trigonometrie tant des triangles plans, que spheriques*. The Hague, 1626, 240 pages (unnumbered). 6.8 × 11.8 cm.

An examination of the copy of this book in the Library of Congress revealed that the main table contains natural sines, tangents, and secants in units of 10^{-5} for every sexagesimal minute of the first quadrant, so arranged that the functions of complementary angles appear on facing pages. Each page contains functions for a range of half a degree.

Following this principal table is a section devoted to the statement and illustration of rules for the solution of the four standard cases of oblique plane triangles. Although Girard frequently resorts to the device of dissecting oblique triangles into right triangles, he does state and use the Law of Sines. In addition, he gives in the form of rules both the Law of Tangents and the Law of Cosines,—the latter in a form involving the versed sine.

A brief section dealing with some general theorems relating to plane polygons is followed by a treatment of both right and oblique spherical triangles. The discussion of the solution of oblique spherical triangles is limited to three cases: (i) all angles given; (ii) all sides given; and (iii) two sides and their included angle given.

The author then gives to five significant figures the length of a side of each of the five regular polyhedra when inscribed in a sphere of diameter 100 000. His results for the regular tetrahedron and regular octahedron contain rounding errors.

A more extensive table is included showing the lengths to the nearest integer of the sides of regular n-gons $[n = 3(1)24]$ inscribed in a circle of diameter 200 000. Careful examination showed this table to be entirely free from error.

After a brief section containing the statement of geometrical problems involving the solution of equations such as $\tan\theta = 2\theta$, $\sin\theta = \theta/\sqrt{2}$, $\theta - \sin\theta = \pi/2$, $\tan 2\theta - \tan\theta = 1$, in modern notation, Girard reproduces Ludolph van Ceulen's 36-figure approximation to π, and presents a tabulation of arc length to radius 10^{10} for the following central angles: 1°(1°)10°-(5°)100°, 150°, 180°, 1′(1′)10′(5′)55′, 1″(1″)10″(5″)55″. This table contains ten rounding errors.

Two rules are stated for the calculation of close approximations to an acute angle of a right triangle in terms of the three sides. In modern trigonometric notation these rules are equivalent to the inequalities:

$$(1) \qquad 3\sin\theta/(2 + \cos\theta) < \theta < \tfrac{1}{3}(2\sin\theta + \tan\theta).$$

The book concludes with miscellaneous information relative to triangles, including a schematic array showing the manner of dependence of the area of a plane triangle on various geometric magnitudes associated with a triangle.

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EDITORIAL NOTE: The inquiring reader will find it of value to read the remarks concerning this work in A. VON BRAUNMÜHL, *Vorlesungen über Geschichte der Trigonometrie*, v. 1, 1900, p. 237–238. It may be remarked that Dr. Wrench's description of Girard's main table is exactly equivalent to that of PITISCUS, 1600 (*MTAC*, v. 3, p. 391).

The problem leading to the equation $\tan\theta = 2\theta$ is as follows (compare N104, LOTHAR COLLATZ, *MTAC*, v. 3, p. 496): What is the arc which equals half its tangent?

A relation differing from (1) only in that the right member is $2\sin\frac{1}{3}\theta + \tan\frac{1}{3}\theta$ was first given by WILLEBRORD SNELL, in his *Cyclometricus*, Leyden, 1621, prop. XXIX, p. 43 f. The first published proof of this right-hand member relation was given by HUYGENS in his *De Circuli Magnitudine inventa*, 1654. If $3y = \theta$ the relation $\theta < 2\sin\frac{1}{3}\theta + \tan\frac{1}{3}\theta$ becomes $y < \frac{1}{3}(2\sin y + \tan y)$, as given by Girard. The single approximation $\theta \approx 3\sin\theta/(2 + \cos\theta)$, was first surmised by NICHOLAS DE CUSA (1401–1464); see his *Opera*, 1514 or 1565, p. 1120–1154. Compare K. T. VAHLEN, *Konstruktionen und Approximationen in systematischer Darstellung*. Leipzig, 1911, p. 188–190.

42. PITISCUS TABLES (Q29, v. 3, p. 398).—In 1927 I purchased for $180, from Wheldon & Wesley, London, a copy of RHETICUS-PITISCUS, *Thesaurus Mathematicus*, Frankfurt, 1613, bound with the 1607 volume (86 p.) described in *MTAC*, v. 3, p. 395. I have also acquired (1924–37) a fine copy of the two volumes (6 parts) of the original 1596 RHETICUS-OTHO, *Opus Palatinum de Triangulis*. With reference to the quality of paper and printing of the 1607 Pitiscus item the inferiority is notably in evidence.

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EDITORIAL NOTE: Among the many other valuable volumes of mathematical and astronomical tables in Mr. Morgan's Library are the following: 1. Ptolemy's Table of Chords, first published in the Liechtenstein edition of the Almagest, 1515; also the George of Trebizond edition of 1528, and the Greek edition of Symon Grynaeus, 1538. 2. Regiomontanus, *Tabulae Directionum*, 1552; also Wittenberg, 1606 edition. These volumes contain a table of sines at interval 1′ and calculated for a radius of 60 000. 3. VIETA, *Canon Mathematicus*. Paris, 1579, formerly the property of CHARLES HUTTON, purchased in 1930 from Sotheran for $200. The great scarcity of this work (of which there is a copy at Brown University) may possibly be due to the fact that on account of errors in the volume, Vieta bought up and destroyed all copies which it was possible for him to locate. 4. B. PITISCUS, *Trigonometry*, 1612 edition (*MTAC*, v. 3, p. 391). 5. H. BRIGGS, *Arithmetica Logarithmica*. London, 1624 (*MTAC*, v. 1, p. 97–98, 170). 6. A. VLACQ, *Arithmetica Logarithmica*. 1628. 7. H. BRIGGS, *Trigonometria Britannica*. London, 1633. 8. H. SHERWIN, *Mathematical Tables*, first edition.

London, 1717. 9. J. DODSON, *Anti-Logarithmic Canon*. London, 1742. 10. G. VON VEGA, *Thesaurus Logarithmorum Completus*. Leipzig, 1794. 11. Astronomical Tables of King Alphonse X of Castile (1203–1284), first printed edition, Venice, 1483; also editions of 1492, 1518, 1524, and Paris, 1545, 1553. 12. E. HALLEY, *Tabulae Astronomicae*. London, 1749. 13. J. CASSINI, *Tables Astronomiques*. Paris, 1740. 14. Mr. Morgan is also the fortunate owner of what appears to be a fifteenth century vellum manuscript of the extraordinarily popular Toledo Planetary Tables, which originated with the astronomer Ibn al-Zarqāla (1029?–1087?). This manuscript consists of 124 leaves measuring 17 × 25 cm. (See G. SARTON, *Introduction to the History of Science*, v. 1, Washington, 1927, p. 758–759.)

CORRIGENDA

V. 1, p. 466, *for* Burden, J. A., *read* Burdon, J. A.

V. 2, p. 384, *for* e *see* Napierian Base, *read* e 54–55, 454; II 15, 68–69, 300; p. 387, *delete* Napierian Base e 54–55, 454; II 15, 68–69; p. 389, *for* Weiertrass, *read* Weierstrass.

V. 3, p. 385, *for* M. R. Hestenis, *read* M. R. Hestenes; p. 466, l. 2, *for* (1.570780, *read* 1.570780; p. 475, l. −12, *for* $Re(S)$, *read* $Re(s)$; p. 496, l. 2, *for* 10^{-5}, *read* $<10^{-5}$.

Index to *MTAC* III: 21-28
1948-1949

The Indices for this volume bring together references to all Articles, RMT, MTE, UMT, ACM, OAC, N, Q, QR, Corrigenda, Subjects, and Names. Mrs. D. H. LEHMER has once more continued her labor of love by preparing the Subject Index, and for this we are deeply grateful. Our octogenarian friend, S.A.J., again has paid special attention to indexing Errata but his notable contribution here and in connection with the whole Index Section is but a tiny fraction of what he has done towards making the volume as a whole more worthy of publication.

HEADINGS FOR INDICES

Numbers issued
Articles
Recent Mathematical Tables
Corrigenda
Mathematical Tables—Errata
Mathematical Tables—Errata, Author Index
Unpublished Mathematical Tables
Automatic Computing Machinery
Other Aids to Computation
Notes
Portraits
Queries
Queries—Replies
Subject Index
Name Index

NUMBERS ISSUED

21, 1948, Jan. (1–68); **22,** April (69–148); **23,** July (149–228); **24,** Oct. (229–332); **25, 1949,** Jan. (333–398); **26,** Apr. (399–452); **27,** July (453–500); **28,** Oct. (501–596).

ARTICLES

F. L. Alt, A Bell Telephone Laboratories' computing machine, 1–13, 69–84
J. Laderman, The square root method for solving simultaneous linear equations, 13–16
H. E. Salzer, Coefficients for expressing the first twenty-four powers in terms of the Legendre polynominals, 16–18
D. F. Ferguson & J. W. Wrench, Jr., A new approximation to π (conclusion), 18–19
W. J. Eckert, The IBM pluggable sequence relay calculator, 149–161
H. F. Mitchell, Jr., Inversion of a matrix of order 38, 161–166
H. E. Salzer, Coefficients for expressing the first thirty powers in terms of the Hermite polynomials, 167–169
A. Fletcher, Guide to tables of elliptic functions, 229–281
A. V. Haeff, The memory tube and its application to electronic computation, 281–286
R. M. Bloch, R. V. D. Campbell & M. Ellis, The logical design of the Raytheon computer, 286–295
J. C. P. Miller, The BAASMTC now RSMTC, 333–340
F. E. Snyder & H. M. Livingston, Coding of a Laplace boundary value problem for the UNIVAC, 341–350
A. Hillman & I. Sherman, Complex zeros of $Y_0(z)$, $Y_1(z)$, and $Y_1'(z)$, 351–352
G. W. King, A method of plotting on standard IBM equipment, 352–355
R. A. Fairthorne & J. C. P. Miller, Hilbert's double series theorem and principal latent roots of the resulting matrix, 399–400
J. O. Harrison, Jr., Piecewise polynomial approximation for large-scale digital calculators, 400–407
H. E. Stelson, The accuracy of linear interpolation in tables of the mathematics of finance, 408–412

F. M. Verzuh, The solution of simultaneous linear equations with the aid of the 602 calculating punch, 453–462

G. D. McCann, The California Institute of Technology electric analogue computer, 501–513

Recent Mathematical Tables

Reviews by numbers, authors, pages. There is no number 607

450 (Mitchell) 19; **451** (Schulze) 19–20; **452** (Müller & Rajna) 20; **453** (Prokeš) 20–21; **454** (Kulikov) 21; **455** (Wijngaarden) 21; **456–457** (Gloden) 21–22; **458** (Gupta) 22–23; **459** (Lahiri) 23; **460** (Mahler) 23; **461** (Robert) 23; **462** (Walfisz) 24; **463** (Ziaud-Din, Kerawala, Kerawala & Hanafi) 24; **464–465** (NBSCL) 25–26; **466** (Reiz) 26; **467** (King) 27–29; **468** (Freeman) 29; **469** (Hage) 29–30; **470** (Ebsen) 31; **471** (Germany, Deutsche Seewarte) 31–32; **472** (Germany, Reichsluftfahrtministerium) 32–34; **473** (U. S. Hydrogr. Off.) 34–36; **474** (Emmons) 36–37; **475** (MIT) 37–40; **476** (Barlow) 84; **477** (Cazzola) 84–85; **478** (Gauss) 85–86; **479** (Comrie) 86–87; **480** (Levallois) 87; **481** (Gloden & Bonneau) 87; **482** (Cracknell) 87; **483** (Rohrer) 88; **484** (NBSCL) 88–89; **485** (Spenceley & Spenceley) 89–92; **486** (Anan'ev) 93; **487** (Marchisio) 93–94; **488** (Great Britain, Naut. Alm. Off.) 94–95; **489** (Duarte) 95; **490** (Ferrier) 95; **491** (Gloden) 96; **492** (Pipping) 96; **493** (Yarden) 96–97; **494** (Zavrotsky) 97; **495** (Salzer) 97–98; **496** (Kelley) 98; **497** (Palm) 98–99; **498** (Blewett) 99; **499** (Bouwkamp) 99–101; **500** (Charney) 102; **501** (Harvard Univ., Comput. Lab.) 102; **502** (Kober) 103; **503** (Magnus & Oberhettinger) 103–105; **504** (Morgan) 105–107; **505** (NBSCL) 107; **506** (U. S. Navy, Off. Res. & Inventions) 107; **507** (Salzer) 107–108; **508** (Dorodnicyn) 108; **509** (Placzek) 108; **510** (Skinner-Truscott) 108; **511** (Tsukhanova & Salamandra) 109; **512** (Khristianovich) 109; **513** (Deutsche Seewarte) 109–110; **514** (Japan, Hydrogr. Dept.) 110–112; **515** (Kopal) 112–114; **516** (Rodrigues) 114–115; **517** (Hickerson) 115–116; **518** (Moretti) 116; **519** (Sachs) 169–170; **520** (Stewart) 170; **521** (Lambert) 170–172; **522** (Peters) 172; **523** (Bosshardt) 172–173; **524** (NBSCL) 173; **525–526** (Rybner) 173–175; **527** (Moon) 175–176; **528** (Beeger) 176; **529** (Kraïtchik) 177; **530** (Lahiri) 177; **531** (Pettit) 178; **532** (Carslaw & Jaeger) 178; **533** (Bell Tel. Labs.) 179; **534** (Charlier) 180; **535** (Cambi) 180–181; **536** (Campbell & Foster) 181–183; **537** (Doetsch) 183–184; **538** (Gray, Merwin & Brainerd) 184–185; **539** (Harvard Univ., Comput. Lab.) 185–186; **540** (Kuznecov) 186; **541** (Lane & Sweeney) 186; **542–543** (NBSCL) 187–188; **544** (King) 188; **545** (MacLane) 188; **546** (Pollard & Present) 188; **547** (Irwin) 189–191; **548** (Tsesevich) 191–195; **549–550** (Japan, Hydrogr. Off.) 195–197; **551** (MIT) 197–198; **552** (Nat. Adv. Comm. for Aeronautics) 199; **553** (Ellis & Walsh) 296; **554** (Gifford) 296–298; **555** (Gloden) 298; **556** (Gupta) 298; **557** (Yarden & Katz) 299; **558** (Akad. N., Matem. Institut) 299; **559** (Birge) 300; **560** (Jackman) 301; **561** (Kelley) 301–302; **562** (Nielsen & Goldstein) 302–303; **563** (Bennett) 303; **564** (Bouwkamp) 303–304; **565** (Cambi) 304–305; **566** (Dwight) 305–306; **567** (Gray & Schelkunoff) 306; **568** (Kaplan) 306–307; **569** (Kourganoff) 307; **570** (Rice) 307–308; **571** (Tietze) 308; **572** (Rice) 308; **573** (Thomas) 308; **574** (Peake) 308–309; **575** (France, Service Technique Aéronautique) 309–311; **576** (Japan, Hydrogr. Off.) 311; **577** (Harvard Univ., Comput. Lab.) 311–314; **578** (Schomann) 355; **579** (Uhler) 355; **580** (France, Inst. Géogr. National) 355–356; **581** (Beall) 356; **582** (Italy, Consiglio Naz. d. Ricerche) 356; **583** (Jordan) 356–357; **584** (Karîâkin) 357; **585** (Anon.) 357; **586** (Gloden) 357; **587** (Kraïtchik) 357–358; **588** (Salzer) 358; **589** (Lienard) 358; **590** (Finney) 359; **591** (Finney & Stevens) 359–360; **592** (Fisher & Yates) 360–361; **593** (Japan, Hydrogr. Dept.) 361–362; **594** (Mainland) 362–363; **595** (Nair) 363–364; **596** (Emde) 364–366; **597** (Bordoni) 366–367; **598** (Great Britain, ACS) 367; **599** (Harvard Univ., Comput. Lab.) 367; **600** (Kitover) 368; **601** (Kober) 368; **602** (Magnus & Oberhettinger) 368–369; **603** (Yuan) 369; **604** (Lecolazet & Pluvinage) 369; **605** (Ugarov) 369; **606** (Guldhammer) 369–371; **608** (Goldschmidt & Pitt) 412; **609** (Zimmermann) 412–413; **610** (Donnay & Hamburger) 413; **611** (Robbins & Smith) 414; **612** (Kerawala & Hanafi) 414; **613** (Salzer) 414–415; **614** (Woodward) 415; **615** (Bose) 415; **616** (Collie, Hasted & Ritson) 415; **617** (Ghizzetti) 415–416; **618** (Goldman) 416; **619** (Greenwood & Miller) 416; **620**

(U. S. Navy, Naval Res. Lab.) 417; **621** (Jaeger) 417; **622** (Zubkov) 417; **623** (Bullard & Cooper) 417–418; **624** (Hay & Gamble) 418; **625** (Herget) 418–419; **626** (Italy, Istituto Idrogr. d. r. Marina) 419–420; **627** (Lapushkin) 420–422; **628** (Merritt) 462; **629** (Mitrinovich) 463; **630** (Sassenfeld & Tschunko) 463; **631** (Comrie) 464–466; **632** (Larsen) 466; **633** (Boll) 466–467; **634** (Wittke) 467; **635** (Vega) 467; **636** (Watson) 468; **637** (Milne) 468–472; **638** (Westberg) 472; **639** (Salzer) 472–473; **640** (Copsey, Frazer & Sawyer) 473; **641** (Kopal) 473; **642** (Rosser) 474; **643** (Deschodt) 474; **644** (Harvard Univ., Comput. Lab.) 474–475; **645** (Macfarlane) 475–477; **646** (Mitchell) 477; **647** (NBSCL) 477–478; **648** (NBSINA) 478; **649** (Stankiewicz) 479; **650** (Arenberg & Levin) 479; **651** (Harvard Univ., Comput. Lab.) 479–482; **652** (Schelkunoff) 482; **653** (Godfrey) 482; **654** (Gröbner & Hofreiter) 482; **655** (Goodwin & Staton) 483; **656** (Stevenson) 483; **657** (NBSCL) 483–484; **658** (Italy, Istituto Idrografico) 513; **659** (Dale) 514; **660** (Sainte-Lagüe) 514; **661** (Danjon) 514–515; **662** (NBSCL) 515–516; **663** (Huskey & Hartree) 516; **664** (Rybner & Sørensen) 516–517; **665** (Harvard Univ., Comput. Lab.) 517–518; **666–667** (Lehmer) 518–519; **668** (Pipping) 519; **669** (Yarden & Katz) 519; **670** (Salzer) 519; **671** (Jeffreys) 519–520; **672** (Smirnov) 520; **673** (Marcum) 521; **674** (Grover) 521; **675** (Hartree & Johnston) 521; **676** (Huckel) 522; **677** (Lowan & Horenstein) 522; **678** (Magnus & Oberhettinger) 522–523; **679** (NBSCL) 523–524; **680** (Bouwkamp) 524; **681** (Levine & Schwinger) 524; **682** (Meixner) 524–526; **683** (Hallén) 526; **684** (LeCaine) 526; **685** (Van de Hulst) 526–527; **686** (Financial Publ. Co.) 527–528; **687** (Italy, Istituto Geogr. Militare) 528–529; **688** (Hoehne) 529; **689** (Azeredo Rodrigues) 529–531; **690** (Sadler) 531.

Corrigenda

P. 67, 147, 227, 332, 398, 452, 499, 563.

Mathematical Tables—Errata

118 (Gudermann) 40–41; **119** (Harvard Univ., Comput. Lab.) 41; **120** (Jahnke & Emde) 41; **121** (Koshliâkov) 41–42; **122** (Spence) 42; **123** (Wrench) 42; **also** Columbia Univ. Press 26, Hage 30, Hayashi 42, Jørgensen 62, Kerawala & Hanafi, 24, King 29, Müller, Rajna & Gabba 20, MIT 38–39, NBSCL 25, Prokeš 20, Reiz 26, Schulze 20, U. S. Hydrogr. Off. 35, Wrench 18–19, Ziaud-Din 24; **124** (Airey) 116–118; **125** (Gloden) 118; **126** (Walther) 119; **127** (Yarden) 119–120; **also** Ageton 145, Anan'ev 93, Bouwkamp 100, Cazzola 84–85, Chebyshev 97–98, Editor 84, l. –5, –6, 106, l. 19–20, Ferrier 95, Fontoura & Penteado 145, Germany, Deutsche Seewarte 138–140, Great Britain, N. A. O. 95, Magnus & Oberhettinger 104–105, NBSCL 88–89, Skinner (Truscott) 108, U. S. Navy 107, Whittaker & Robinson 98; **128** (Airey) 199, **129** (Becker & Van Orstrand) 200–201, **130–131** (Dwight) 201; **132** (Gauss) 201–203, **133** (NBSCL) 203; **134** (Okaya) 203; **135–137** (Peters) 203–205; **also** Cajori 224, Cambi 181, Doetsch 183, Felkel 171, Ferrari 194, Kraïtchik 177, Lambert 171–172, Lane & Sweeney 186, Lehmer 227, Moon 175, Peters 172, Pettit 178, Russell & Shapley 192, Rybner 174, Tsesevich 193; **138** (Adams) 314; **139** (Comrie) 314; **140** (Hardy & Wright) 314; **141** (Jahnke & Emde) 314–315; **142** (NBSCL) 315; **143** (Peters) 315; **144** (U. S. Hydrogr. Off.) 315–316; **also** Airey 261, Akushskiĭ & Ditkin 299, Bertrand 261–262, Dale 262, DeMorgan 308, Ditkin 299, Dwight 262, FMR 276–277, France 309, Gauss 262, Gifford 297, Glaisher 263, Gossot 263, Greenhill 263, Hammer 308, Hancock 263, Harvard Univ., Comput. Lab. 313, Hayashi 263–266, Heuman 266, Hippisley 266–267, Innes 266–267, Jahnke & Emde 267–268, Legendre 268–274, Lévy 274–275, Lîûsternik 299, Loxhay 280, Meissel 275–276, Merfield 269, Moore 276, Nagaoka & Sakurai 276, Pearson 306–307, Plana 277–278, Potin 278, Rosenbach, Whitman & Moskovitz 278, Runkle 278, Samoĭlova-Îakhontova 278–279, Schlömilch 279, Soldner 308, Spenceley 280, Verhulst 280–281, Wayne 281, Weigand 308, Yarden & Katz 299; **145** (Becker & Van Orstrand) 371; **146** (Fisher) 371; **147** (Kraïtchik) 372; **148** (Oughtred) 372; **also** France, Inst. Géog. Nat. 356, Guldhammer 371, Jahnke & Emde 365–366, Kariâkin 357, Pitiscus 391–392, Rheticus 395–396, Ugarov 369, Uhler 355; **149** (Adams) 423; **150** (Glazenap) 423;

151 (Jahnke & Emde) 423; **152** (Kelley) 423; **153** (Richmond) 423–424; **154** (BAASMTC) 424; **155** (U. S. Coast and Geod. Survey) 424–425; **also** Bertrand 449, Buckingham 413, Callet 426, Davis 424, Davis & Kirkham 450, Goldman 416, Gray & Mathews 449–450, Hay & Gamble 418, Herget 419, Kerawala & Hanafi 414, Lapushkin 422, Meissel 449–450, U. S. Navy 417, Zimmermann 412; **156** (Holtappel) 485; **157** (Kelley) 485; **158** (Pearson) 485; **159** (U. S. Coast and Geod. Survey) 485; **also** Boll 466–467, Cunningham 486, Hacker 493, NBSINA 478, Newman 477, Powell 477, Seewald 472; **160** (D. H. Lehmer) 531; **161** (D. N. Lehmer) 531–532; **also** Boll 518, Bouwkamp 525, Briggs 516, Dale 514, Feller 520, FMR 551, Gauss 551, Hallén 526, Huckel 522; Lehmer 518–519; Magnus & Oberhettinger 523, Meixner 525, Rheticus 556–557, Watson 549–550.

Mathematical Tables—Errata, Author Index

Adams 314, 423
Ageton 145
Airey 116–118, 199, 261
Akushskiĭ & Ditkin 299
Anan'ev 93
Becker & Van Orstrand 200–201, 371
Bertrand 261–262, 449
Boll 466–467, 518
Bouwkamp 100, 525
Brandenburg 43
BAASMTC 424
Briggs 516
Buckingham 413
Cajori 224
Callet 426
Cambi 181
Cazzola 84–85
Chebyshev 97–98
Columbia Univ. Press 26
Comrie 314
Cunningham 486
Dale 262, 514
Davis 424
Davis & Kirkham 450
DeMorgan 308
Ditkin 299
Doetsch 183
Dwight 201, 262
Editor 84, 106
Felkel 171
Feller 520
Ferrari 194
Ferrier 95
Fisher 371
FMR 276–277, 551
Fontoura & Penteado 145
France 309, 356
Gauss 201–203, 262, 551
Germany, Deutsche Seewarte 138–140
Gifford 297
Glaisher 263
Glazenap 423
Gloden 118
Goldman 416
Gossot 263
Gray & Mathews 449–450
Great Britain, N.A.O. 95
Greenhill 263
Gudermann 40–41
Guldhammer 371
Hacker 493
Hage 30
Hallén 526
Hammer 308
Hancock 263
Hardy & Wright 314
Harvard Univ., Comput. Lab. 41, 313
Hay & Gamble 418
Hayashi 42, 263–266
Herget 419
Heuman 266
Hippisley 266–267
Holtappel 485
Huckel 522
Innes 266–267
Jahnke & Emde 41, 267–268, 314–315, 365–366, 423
Jørgensen 62
Kariâkin 357
Kelley 423, 485
Kerawala & Hanafi 24, 414
King 29
Koshliâkov 41–42
Kraĭtchik 177, 372
Lambert 171–172
Lane & Sweeney 186
Lapushkin 422
Legendre 268–274, 306–307
Lehmer, D. H. 227, 518–519, 531
Lehmer, D. N. 531–532
Lévy 274–275
Lûsternik 299
Loxhay 280
Magnus & Oberhettinger 104–105, 523

MIT 38–39
Meissel 275–276, 449–450
Meixner 525
Merfield 269
Moon 175
Moore 276
Müller, Rajna & Gabba 20
Nagaoka & Sakurai 276
NBSCL 25, 88–89, 203, 315
NBSINA 478
Newman 477
Okaya 203
Oughtred 372
Pearson 306–307, 485
Peters 172, 203–205, 315
Pettit 178
Pitiscus 391–392
Plana 277–278
Potin 278
Powell 477
Prokeš 20
Reiz 26
Rheticus 395–396, 556–557
Richmond 423–424
Rosenbach, Whitman, & Moskovitz 278
Runkle 278
Russell & Shapley 192
Rybner 174
Samollova-Ťakhontova 278–279
Schlömilch 279
Schulze 20
Seewald 472
Skinner (Truscott) 108
Soldner 308
Spence 42
Spenceley 280
Tsesevich 193
Ugarov 369
Uhler 355
U. S. Coast & Geod. Survey 424–425, 485
U. S. Hydrogr. Off. 35, 315–316
U. S. Navy 107, 417
Verhulst 280–281
Walther 119
Watson 549–550
Wayne 281
Weigand 308
Whittaker & Robinson 98
Wrench 18–19, 42
Yarden 119–120
Yarden & Katz 299
Ziaud-Din 24
Zimmermann 412

Unpublished Mathematical Tables

There is no number 66

63 (Wrench) 42–43; **64** (Porter) 43; **65** (NBSCL) 43–44; **67** (Poulet) 120; **68** (Salzer) 120–121; **69** (Salzer) 205; **70** (Spenceley) 205; **71** (Salzer) 205–206; **72** (Salzer) 316; **73** (Dixon) 316–317; **74** (Salzer) 317; **75** (Univ. California, Statist. Lab.) 372–373; **76** (Faucher) 425; **77** (Bower) 425–426; **78** (Greenwood) 426; **79** (Bartol Res. Foundation) 485; **80–81** (Gloden) 486; **82** (Poletti) 532; **83** (Salzer) 532.

Automatic Computing Machinery

Technical Developments: See under Articles, Alt; Bloch, Campbell & Ellis; Eckert; Haeff; Harrison & Malone; McCann; Snyder & Livingston; Verzuh.

Discussions: S. Lubkin, Decimal point location in computing machines, 44–50; J. L. McPherson, Applications of large-scale high-speed computing machines to statistical work, 121–126; H. F. Mitchell, Jr., Inversion of a matrix of order 38, 161–166 [also as an Article]; R. F. Clippinger, Airflow problem planned for the ENIAC, 206–207; B. L. Hicks & H. G. Landau, Nonlinear parabolic equations, 207–208; J. V. Holberton, Laminar boundary layer flow in a compressible fluid, 208; J. H. Levin, On the approximate solution of a partial differential equation on the differential analyzer, 208–209; M. Lotkin, Computation of the airflow about a cone cylinder, 209–210; R. M. Bloch, R. V. D. Campbell & M. Ellis, General design considerations for the Raytheon computer, 317–323; Dorrit Hoffleit, A comparison of various computing machines used in the reduction of Doppler observations, 373–377; Florence Koons & S. Lubkin, Conversion of numbers from decimal to binary form in the EDVAC, 427–431; E. F. Moore, A new general method for finding roots of polynomial equations, 486–488; J. L. Meriam, Procedure for the machine or nu-

merical solution of ordinary linear differential equations for two-point linear boundary values, 532–539.

Bibliography: Burks, Goldstine & Neumann; Comrie; Goldstine & Neumann; Hartree, 50–57. Anon.; Burks, Goldstine & Neumann; Cesareo; Hartree; Juley; Moore School of Electr. Engineering, 126–132. Aiken & Hopper; Anon.; Berkeley; Burks; Comrie; Condon; Curry & Wyatt; Koehler; Russo; Seeber; Lilley; Tjänste; Tumbleson, 210–213. Burks; Djure & Welin; Goldstine; Great Britain, Post Office Dept.; Huskey; IBM, 323–326. Aiken & Hopper; Anon.; Bigelow, Pomerene, Slutz & Ware; Couffignal; Haeff; Lion; Moore School Electr. Engin.; Inst. Radio Engin., 377–380. Brillouin; Eckert; Fuller; Harvard Univ., Comput. Lab.; Inst. Adv. Study; Irwin; Korn; MacLeish, O'Neal & Tyler; Moore School Electr. Engin.; O'Neal & Tyler; Williams & Kilburn; U. S. Air Force, 431–445. Booth & Britten; Cohen & Keye; Coombs; Hamilton; Hoberg & Ulman; Page; Patterson, Snyder, Tabor & Travis; Pfeiffer; Wilkes; Wilkes & Renwick, 488–490. Anon.; Arenberg; Boothroyd & Cherry; Brainerd & Sharpless; Bronzo & Cohen; Chu; Davis; Goldstine & Neumann; Hartree, Newman, Wilkes, Williams, Wilkinson & Booth; McCool; Sharpless; Wiener, 539–544.

News: Amer. Acad. Arts Sci. 216, Amer. Phys. Soc. 326–327, Amer. Statist. Assoc. 134, Assoc. Computing Machin. 57, 132–134, 216, 380, 445–446, 490, 544–545, Automatic relay computer 216, BAAS 132, Electronic digital computing in England 213–216, Institute Radio Engins. 491–492, Institute Teachers Math. 380, Institution Electr. Engins. 57, IBM 381, IBM Selective Sequence Electronic Calculator 216–217, Intern. Sci. Radio Union 327, Mass. Inst. Techn. 492, Math. Assoc. Amer. 58, Math. Computing France 58, NBSINA 545–547, NBSMDL 327–328, Nat. Electronics Conference 134, Naval Res. Lab. 446, Office Naval Res. 446, Personal 328, Swedish State Board Computing Machinery 492; Symposia on modern Calc. Mach. 381–388.

Other Aids to Computation

G. R. Stibitz, A new class of computing aids 217–221.

G. R. Stibitz, Electric root-finder 328–329.

Bibliography: Akushskiĭ; Bartlett; Baudouin; Bergman; Berry & Pemberton; Bruk; Eckert; Fürth & Pringle; Globe-Hilsenrath; Herget; Herrick; Kind-Schaad; King; Renner; Reynolds; Shaffer, Schomaker & Pauling; Tolstov, 58–61. Anon.; Baracket; Benjamin; Black & Olds; Booth; Great Britain, Ministry of Supply; Harbert; Hooks; Johnson; Lafferty; Manley; Meyer zur Capellen; Psychometric Soc. & IBM; Vajda; Waddington; Weissenstein, 134–138. Adcock; Akushskiĭ; Anon.; Gavrilko; Leĭbin; Packh; Weems, 221–222. Babcock; Goldberg & Brown; Murray; Tiller, 329–330. Camp; Carle; Comrie; Gilbert; Othmer, Josefowitz & Schmutzler; Purdie; Redheffer; Richardson; Tea; Wilkas; Wittke, 388–390. Bothel; Goldberg; Hardy & Dench; Lo-Ho; Nachod; Rubbert; Strohm & DeGroot; Svoboda, 446–448. Hacker; Kiely; Reinhardt; Willers, 492–494. Brown; Carter & Sadler; Knauss; Richards & Sadler; Vand; Wittke, 548–549.

Notes

80. R.C.A., Certain gear-ratio tables, 61; 81. R.C.A. & M. S. Corrington, *Guide* (*MTAC* no. 7), 62; 82. W. F. Willcox, Herman Hollerith, 62–63; 83. H. E. Salzer, Interpolation of functions tabulated at fixed points, 63–64; 84. The National Applied Mathematics Laboratories, 64–65; 85. NAML, 65; 86. Russia and mathematics, 65–66; 87. J. B. Parker & D. H. Sadler, German altitude and azimuth tables," 138–141; 88. R.C.A., Harry Bateman, 141–142; 89. H. S. Uhler, The magnitude of higher terms of the Lucasian sequence, 142–143; 90. *Mathematical Table Makers*, 143–144; 91. R.C.A., Portraits of table makers, 144–145; 92. C. H. Smiley, Portuguese navigation tables, 145; 93. A film of part of Kulik's Magnus Canon for sale, 222; 94. R.C.A., Lambertian or lambda function, 222–225; 95. R.C.A., Rydberg interpolation table, 330; 96. R.C.A., Bartholomäus Pitiscus (1561–1613), 390–397; 97. Fritz Emde, 397–398; 98. R.C.A., Mersenne numbers, 398; 99. R.C.A., Daniel Friedrich

Ernst Meissel, 449–451; **100.** A. Ferrier, A new factorization of $2^n + 1$, 451; **101.** R.C.A., Newman's *Mathematical Tracts*, 451; **102.** Tables of $x \tan x$, 451; **103.** Editors, Arnold Noah Lowan, 494–495; **104.** L. G. Pooler, Roots of certain transcendental equations, 495–496; **105.** A. Ferrier, Note on the factors of $2^n + 1$, 496–497; **106.** Editors, George Neville Watson —table maker, 549–550; **107.** D.H.L., The lemniscate constant, 550–551; **108.** R.C.A., Newman and George Eliot 551–552; **109.** R.C.A., Rheticus, with special reference to his Opus Palatinum, 552–561.

Portraits

F. Emde, no. 25; A. N. Lowan, no. 27; D. F. E. Meissel, no. 26; G. N. Watson, no. 28.

Queries

24. C. W. Horton, Integral of a Struve function, 66; **25.** R.C.A., Russian Bessel function tables, 66; **26.** R.C.A., Some clothoid or Euler spiral tables, 146; **27.** R.C.A., First table of $\tan^{-1} x$, 226–227; **28.** A. Fletcher, Moseley on definite integrals, 331; **29.** R.C.A., Pitiscus tables, 398; **30.** R.C.A., Girard and Snell tables, 451–452; **31.** R.C.A., Japanese World War II editions of mathematical tables, 498; **32.** French and Russian translations of a Vega-Bremiker table, 561.

Queries—Replies

There is no no. **36.**

33. J. Ginsburg, Editor, Portraits and biographies of British mathematical table makers, 66–67; A. J. Thompson, Henry Briggs, 67; **34.** R.C.A., An Engel table, 146; **35.** J. Todd, Tables of $\tan^{-1} (m/n)$, 227; **37.** N. G. W. H. Beeger, Hendrik Anjema, 331–332; **38.** Log log tables, 398; **39.** R.C.A., Some clothoid or Euler spiral tables, 452; **40.** A. Fletcher, Mrs. G. O. Savage, Editor, Pitiscus tables, 498–499; **41.** J. W. Wrench, Jr., Girard and Snell tables, 561–562; **42.** W. D. Morgan, Editor, Pitiscus tables, 562–563.

Subject Index

Accounting 389, 539, 548
Acoustics 37, 134–135, 416, 540
Actuarial Tables 466
Addition and Subtraction Logs 20, 85, 513
Aerodynamics *see* Fluid Mechanics
Air Navigation *see* Navigation
Airy's Integral 337, 339, 418
Algebraic Equations 13–16, 453–462, 469, 486–488
Algebraic Equations, Mechanical Solutions of 60–61, 81, 137–138, 221, 299, 329–330, 436, 494, 545
Aliquot Series 120
Alpha Functions 112–114
Altitude-Azimuth 31–36, 59, 110–112, 114–116, 138–141, 309–311, 530
Analogue Machines 56–57, 61, 127, 208–209, 212, 217, 323, 327–329, 382, 441, 491–494, 501–513, 539–540, 542–544
Anger Functions 103, 365–366
Annuity 29–30, 408–412, 465–466
Antenna Theory 303–304, 524
Antilogarithms 84–85, 463, 465, 514
Arc Cosine, etc., *see* Cosine, Inverse, etc.
Arc Length 20, 85, 94, 171, 463, 513, 562
Astronomy 21, 31–32, 85, 109–116, 189–195, 418–419, 440–441, 513, 526, 531, 556
Astrophysics 526–527
Automatic Computing Engine (ACE) 214–215
Automatic Relay Computer (ARC) 216
Automatic Sequence Controlled Calculator (ASCC) 54–56, 128, 130–131, 149, 210, 213, 314, 377–378, 493, 518
Azimuth Computer 58–59

Babbage's "Engines" 53–54, 210, 215, 432, 493
Ballistics 37–40, 127, 197–199, 252–258, 311–313, 373–377, 437
Bell Telephone Relay Computer 1–13, 54, 69–84, 207–208, 210, 375–377, 433
Bernoulli Functions 368
Bernoulli Numbers 514
Bessel Functions
  Asymptotic Expansions 335, 365
  Combinations of 305–306, 367, 415, 417
  Integrals of 303, 308, 340, 417–418, 550

Logarithms of 62
Modified 105
of Complex Argument 25, 333, 335, 352
of First Kind and Integral Order 41, 65, 88, 102–103, 107, 116–117, 180–181, 183, 185–186, 199, 253, 333, 335, 337, 339, 366–367, 416, 449–450, 474–475, 514, 549
of Fractional Order 187, 339, 466–467, 484, 514, 550
of Purely Imaginary Argument 105–107, 365–366, 415, 478, 514, 549
of Second Kind and Integral Order 107, 187–188, 203, 315, 333, 549
Spherical 26
Turning Points of 26
Zeros of 26, 102, 305–306, 340, 351–352, 366, 496, 514, 549–550
Bessel Interpolation Coefficients 466
Beta Function 485
Binary Computers 48, 128, 287–288, 379, 427–431, 441–443, 488, 491–492, 540
Binary Quadratic Forms 22–23, 43
Binomial Coefficients 87, 171, 339, 463, 472, 514
Binomial Congruences 335
Binomial Distribution 179
Biography and Bibliography, Portraits: Airey 143–144, Anding 143, Anjema 331–332, Babbage 143, Barlow 66–67, Bateman 141–142, Bauschinger 143, Becker 143, Bessel 143, Bierens de Haan 143, Borda 143–144, Briggs 67, Brown 143, Bürgi 143–144, Burrau 143, Charlier 180, Chebyshev 144, Cohn 143, Comrie 143–144, Cunningham 143–144, Dase 143, Davis 143, Dickson 143, Dodson 67, Dwight 143, Eckert 145, Eliot, George 551–552, Emde 144, oppo. 333, 397–398, Farley 66, Filipowski 66–67, Gauss 85–86, Gifford 296–297, Glaisher (J.) 143, Glaisher (J. W. L.) 143–144, Gray 66–67, Hoëne-Wroński 143, Hollerith 62–63, Hoppe 143, Hutton 143, Jacobi 143, Kepler 143–144, Kraïtchik 143, Kruse 144, LaLande 143, Lambert 145, Legendre 143, Lehmer (D. H.) 143–144, Lehmer (D. N.) 143, Lodge 143, Lohse 143, Lommel 143, Lowan 143, oppo. 453, 494–495, Markov 143, Martin 143, Meissel oppo. 399, 449–451, Miller 143, Müller 20, Napier 143, Newman 551–552, Nielsen 143, Ostrogradskiĭ 145, Oughtred 372, Pearson 143, Peirce 143, Peters 143–144, Pitiscus 390–397, 498–499, 562, Rheticus 552–562, Rivard 143, Rydberg 330, Sang 143, Segner 226, Sharp 143, Sheppard 143, Stevin 143–144, Stieltjes 143, Tallqvist 143, Thompson 143, Turner 143, Uhler 143, Viète 143, Watson oppo. 501, 549–550, Wright 223–225, 372. *Also* 50–57, 58–61, 126–132, 134–138, 210–213, 221–222, 250–260 (Elliptic Functions), 323–326, 329–330, 377–380, 388–390, 431–445, 446–448, 488–490, 492–494, 539–544, 548–549
Bivariate Interpolation 524
Bivariate Normal Distribution 372–373, 474
Bond Values 29–30, 408–412, 527–528

Calculating Machines *see* Computing Machines
CIT (Cal. Inst. Techn.) Electronic Analogue Computer 501–513
Chebyshev Polynomials 17, 64, 103, 119–121, 300, 405–407, 471
Chebyshev Quadrature 97
Chi Square Distribution 302, 363, 466, 515, 519–520
Christoffel Numbers 26, 41, 416, 473
Class Number 22
Clausen Integral 368
Clothoid 146, 452
Coding 48–51, 54–56, 78–79, 320, 444, 488–490, 541–542, 545
Complex Numbers 493, 516–517
Computational Methods 131–132, 216, 299, 381–382, 400–407, 434–440, 443, 468–472, 486–488, 493–494, 532–539, 545–547
Computing Machines 1–13, 44–58, 69–84, 121–138, 149–161, 206–222, 286–295, 299, 317–330, 373–382, 388–390, 426–448, 486–492, 501–513, 539–548
Cone Function 103, 105
Conformal Mapping 103, 250, 368, 493, 545–546
Congruences 96, 177, 357, 486
Constants 20, 42–43, 94, 254, 302, 369, 463, 466–467, 478, 513–515
Continued Fractions 96, 519
Conversion 20, 94, 340, 463, 465–467, 513–515
Coordinates, Polar 94, 340
Correlation 60, 371, 389
Cosecant, Log of 19, 371, 513
Cosecant, Natural 19, 93–94, 172, 371, 513–515
Cosecants, Sums of 549–550

Cosine, Hyperbolic 86–87, 93, 170–171, 205, 336, 463, 465–466, 514
Cosine, Hyperbolic Inverse 517–518
Cosine, Inverse 85, 87, 463, 465
Cosine, Log of 19–20, 85–87, 355–356, 370, 463–467, 513–514
Cosine, Natural 19–20, 85–87, 93–94, 146, 172, 370, 413, 425–426, 463, 466–467, 485, 513–516
Cosine Integral 104, 336, 338, 417, 479–482, 514, 526
Cosine Integral, Iterated 526
Cotangent, Hyperbolic 88–89, 296
Cotangent, Inverse 85, 87, 465
Cotangent, Log of 19–20, 85, 355–356, 371, 463–467, 513
Cotangent, Natural 19–20, 85, 88–89, 93–94, 172, 296, 371, 463, 466–467, 513
Crystallography 60, 136, 413, 540
Cube Root 19, 84–85, 98, 302, 340, 370, 463, 466, 494, 513–514
Cubes 19, 84–85, 171, 370, 463, 466, 513–514
Cubes, Endings of 298
Cubic Equations 171, 388, 467
Cubic Forms 24
Cuboids 177
Cybernetics 543–544
Cylinder Functions 103–104, 369
Cylinder Functions, Elliptic 101
Cylinder Functions, Parabolic 104, 365

Decimal Equivalents of Fractions 19, 462, 514
Derivatives, Numerical 414–415
Desk Computers 136–137, 493–494, 548–549
Determinants 469
Difference Engine *see* Babbage's "Engines"
Difference Equations 471
Differential Analyzer 54, 56–57, 127, 137, 208–209, 212, 491, 493
Differential Equations 61, 81, 132–133, 160, 184–185, 206–210, 304–306, 330, 436–437, 470, 494, 501–507, 511, 522, 524–526, 532–540, 548
Differentiation, Numerical *see* Numerical Differentiation
Digamma Function 472
Digital Machines 1–13, 44–58, 69–84, 121–134, 149–161, 206–208, 210–217, 286–295, 317–328, 373–381, 427–446, 488–492, 540–542, 544–548
Diophantine Equations 95, 177, 333
Divisors 334, 337, 339
Dixon Function 229, 249–250
Doppler Method 373–377
Duodecimal Tables 23, 357, 388

Eigenvalues 524–526
Elasticity 29, 93, 357, 368, 483, 511, 545
Electrodynamics 369
Electromagnetism 237–239, 250, 253, 256, 308–309, 483–484
Electronic Delay Storage Automatic Calculator (EDSAC) 490, 542
Electronic Discrete Variable Computer (EDVAC) 54, 57, 129–131, 213, 379–380, 382, 427–431, 437, 440–441, 444–445, 489–490, 540–541
Electronic Numerical Integrator and Computer (ENIAC) 1–4, 7, 53–54, 57, 83–84, 126, 129, 131, 133, 185, 206–208, 210, 212–213, 323–326, 330, 375–377, 433, 437, 440, 446, 493, 518, 540–541
Elliptic Functions 89–92, 104, 229–281, 334–335, 365, 369, 449, 514, 521, 551
Elliptic Functions, Cylinder *see* Cylinder Functions, Elliptic
Elliptic Integrals 27–28, 89, 99, 104, 229, 231–281, 304–307, 369
Emden Functions 336, 339
Equation Solvers 61, 137–138, 221, 323, 329–330, 447, 491
Equations, Algebraic *see* Algebraic Equations
Equations, Transcendental *see* Transcendental Equations
Error Function, *see* Probability Integral
Euler Numbers 467, 514
Euler Product 340
Everett Interpolation Coefficients 187, 339, 366, 418–419, 470, 472, 477–478
Exponential Function 85, 87, 103, 173, 334, 371, 426, 463, 465–466, 485, 514
Exponential Integral 104, 336, 338, 514, 526
Exponents 176, 357

Factor Tables (general) 85, 87, 95, 171–172, 222, 334, 336, 339, 462–463, 466–467, 532
Factor Tables (special) 21, 23, 95, 118–119, 299, 357, 451, 486, 496, 518–519
Factorials 85, 205, 340, 355, 357, 466
Factorials, Logs of 85, 466, 514
Factorials, Ratios of 26
Factorials, Reciprocals of 514
Farey Series 333, 339
Fermat's Converse 314, 451, 496–497, 518–519
Fermat's Last Theorem 493
Fibonacci Sequence 96–97, 119–120, 299
Figurate Numbers 171
Fisher's z-Test 519–520, 524
Fluid Mechanics 36–40, 109, 116, 136, 197–199, 206–207, 209–210, 299, 311–313, 356, 369, 511, 522, 545

Formulae 94, 251, 369, 466, 474, 513–515, 521–523
Fourier Coefficients 241, 251, 413, 471
Fourier Integrals 21, 181–183, 475–476
Fourier Series 16, 82, 251, 434
Fourier Transformation 104
Fourier Transformer 61
Fresnel Integrals 104, 146, 336, 417, 467, 479
Functions (special) *see* under Alpha, Anger, Bernoulli, Bessel, Beta, Cone, Cylinder, Digamma, Dixon, Elliptic, Emden, Exponential, Gamma, Hankel, Harmonic, Hermite Probability, Hyperbolic, Hypergeometric, Intensity, Jacobi Zeta, Kelvin, Kummer, Laguerre, Lambda, Legendre, Lommel, Lommel-Weber, Mathieu, Planck's, Polygamma, Psi, Radiation, Ramanujan's, Riemann Zeta, S and T, Scattering, Spherical, Spheroidal Wave, Stokes', Struve, Symmetric, Tetragamma, Theta, Toroidal, Trigamma, Trigonometric, Weber, Wedge, Weierstrass Elliptic, Whittaker, Zeta.

Galois Fields 97
Games 389, 490–491
Gamma Function 98, 103, 307, 317, 336, 338, 369, 466, 472, 478, 514, 521, 549, 551
Gamma Function, Complex Argument 415–416
Gamma Function, Incomplete 104, 302
Gamma Function, Log of 340, 465, 549
Gaussian Quadrature 97, 470
Gear Ratios 61, 462
Geodesic Tables 86–87, 513, 528–529
Grades 94
Graeffe Process 487–488, 494
Graphs 138, 299, 306, 388
Gravitation 20, 417–418
Gregory-Newton Interpolation Coefficients 358, 470
Gudermannian 466, 514

Hankel Functions 299
Harmonic Analysis 61
Harmonic Analyzer 137–138
Harmonic Functions 61, 182
Harmonic Series 159–160
Harmonic Syntheses 413
Heat Equation 109, 240, 299, 511
Hermite Polynomials 103, 167–169, 180, 416, 471, 473, 521
Hermite Probability Function 338
Hydraulics, Hydrodynamics, *see* Fluid Mechanics
Hyperbolic Functions 371, 517–518
Hypergeometric Function 103–104, 248–249, 307, 338, 369, 522
Hypergeometric Function, Confluent 102, 336, 340, 366, 369, 520, 522–524

Ideals, Reduced 336, 339
Indices 372
Integral Equations 104, 390
Integrals, Misc. Definite 21, 29, 188, 307–308, 331, 369, 418, 472–474, 479–482, 524, 526–527
Integrals, Multiple 479, 550
Integrals, Tables of 87, 188, 251, 466, 482, 514
Integraph 138, 306, 390
Integration, Numerical *see* Numerical Integration
Integration of Differential Equations *see* Differential Equations
Intensity Function 484
Interest, Compound 408–412, 465–466, 514, 528
International Business Machines (IBM) Selective Sequence Electronic Calculator 216–217, 326–328, 330, 367, 381–382, 540
International Business Machines (IBM) Sequence Relay Calculator 83, 133, 149–161, 207–208, 375–377
Interpolation 26, 63–64, 82, 85, 205–206, 212, 330, 339, 358, 361–362, 366, 408–412, 414–415, 418–419, 466, 468–472, 481, 494, 515, 519
Isograph 138, 491

Jacobi Nome q 229, 234–235, 256
Jacobi Polynomials 103
Jacobi Zeta Function 90–91, 241, 244–246, 256–257
Julian Year 514

Kelvin Function 340
Kummer Function 104

Lagrangean Coefficients 98, 302, 414, 466, 469, 472
Lagrangean Polynomials 469
Laguerre Functions 104, 365
Laguerre Polynomials 103, 532
Lambda Function, *see* Lambertian
Lambertian 170, 222–225
Laplace Coefficients 229, 237, 252, 470
Laplace Equation 106, 239, 299, 343–346
Laplace Transforms 104, 178, 183–184, 476, 523
Lattice Points 22–23
Least Squares 189–191, 300, 302–303, 470–471, 513

Legendre Functions 103, 252, 365, 525
Legendre Polynomials 16–18, 99, 103, 167, 186, 334, 337, 339, 369, 470–472, 484, 514, 521
Lemniscate Constant 550–551
Linear Equations 13–16, 60–61, 81, 436, 453–462, 545
Linear Forms 21–22
Linkages 222, 447–448
Logarithmic Integral 104, 308
Logarithms, Common 19–20, 84–85, 87, 340, 355, 365, 370, 425–426, 463, 465, 467, 513–514
Logarithms, Natural 85–87, 142–143, 171, 360, 372, 463, 465, 513–514
Logarithms, Addition and Subtraction *see* Addition and Subtraction Logarithms
Logs of Primes 463
Log Log 398, 426, 548
Lommel Functions 103, 186
Lommel-Weber Functions 103, 336

M, the Modulus of Common Logarithms 20, 85, 465
Magic Squares 531–532
Magnetism 99, 237–239
Mark I 58, 69, 79, 83, 432–446, 493, 545
Mark II 2, 58, 83–84, 210, 433–434, 437–438, 446, 493
Mark III 446, 492, 499, 540
Mathieu Functions 103–104, 366
Matrices 159–166, 399–400, 436–437, 453–456, 469, 473, 544
Mellin Transform 475–476
Memory Tube 281–286
Mersenne Numbers 142, 398
Meteorology 85, 102
Monte Carlo Problem 544, 546–547
Mortality Tables *see* Actuarial Tables
Motor-boating 508
Multiplication Tables 299, 355, 465

Navigation 32–36, 43–44, 58–59, 138–141, 145, 195–197, 221, 309–311, 361–362, 369–371, 419–422, 529–531
Network Analyzer 222, 417, 539
Newton Interpolation Coefficients 470, 472
Nomographs 135–137, 173–175, 390, 447, 514, 529–531, 547
Normal Distribution 515
Normal Ordinate 466
Number Theory 21–24, 95–97, 142–143, 176–177, 298–299, 333–340, 357–358, 451, 468, 486, 496–497, 518–519, 531–532, 550–551
Numerical Differentiation 466, 468–470, 494
Numerical Integration 60, 81–82, 97, 107–108, 132, 466, 468–470, 494, 516
Numerical Integration of Differential Equations *see* Differential Equations
Numeroscope 432

Optics 99–101, 412, 482, 524
Opus Palatinum 554–558
Orthogonal Polynomials 300, 360, 369, 472

Partitions 340, 519, 550
Pearson Integral 335
Pell Equation 335
Physical Tables 85, 253–254, 513
Physics 20, 107–109, 175–176, 237–239, 250, 253, 256, 308–309, 369, 483–484, 522–523, 547
Pi 18–19, 20, 172
Pi, Multiples of 85, 463, 513
Pi, Powers of 42–43, 85
Planck's Function *see* Radiation Function
Planimeter 138
Poisson Distribution 301
Poisson-Laplace Equation 61
Polar Coordinates 172–173
Polygamma Function 334, 336
Polygons 561
Polyhedra, Regular 467, 561
Polynomial Approximations 400–407, 426
Powers, Fifth, Decomposition into 336, 339
Powers, Fourth 87, 466, 514
Powers, Fractional 466, 485
Powers, Higher than Fourth 85, 167–169, 171, 334, 337, 339, 463, 466, 514
Powers, Negative 466, 485
Powers, Sums of 358, 514
Prime Pairs 532
Primes, Distribution of 532
Primes, Lists of 85, 87, 171–172, 357, 467, 532
Primes, Logs of 463
Primes, Number of 95
Primes of Special Forms 119, 357–358
Probability Integral 87, 98, 178, 180, 301–302, 316–317, 336–337, 339, 359–360, 365, 466, 474, 514
Probability Integral, Integrals and Derivatives of 336, 338, 521
Probability Integral, Inverse of 180, 301
Psi Function, *see* Digamma Function
Punch Card Machines 2–4, 7, 60, 135, 150–151, 221, 325, 352–355, 376–377, 381, 433
Punch Card Methods 27–29, 60, 135, 255, 352–355, 456–462
Punch Card Tables 60

Quadratic Forms 22–23, 43
Quadrature *see* Numerical Integration
Quantum Mechanics 27–29, 188
Quarter Squares 514
Quartic Congruences 21, 96, 486

Radiation 107, 484
Radiation Function 175–176
Radioactivity 548
Railway Transition Curves *see* Clothoid
Ramanujan's Function $\tau(n)$ 23, 177, 298, 468, 550
Random Numbers 547
Raytheon Computer 286–295, 317–323, 327, 540
Reciprocals 84–85, 87, 94, 169–170, 466, 513
Reciprocals of Factorials *see* Factorials, Reciprocals of
Recurring Series 142, 519
Reeves Electronic Analogue Computer (REAC) 501
Reeves Instrument Corporation Analogue Computer (REEVAC) 327, 382
Relay Computers 1–13, 54, 69–84, 127, 149–161, 164–166, 207–208, 216–217, 376–378, 545
Riemann Zeta Function 21, 366, 475–476
Root-finder 328–329
Roots, Higher than the Third 340, 466
Rounding-off Errors 437, 544

S and T Functions 20, 85–86, 464–465, 467, 515
Scattering Function 107, 484
Secant, Inverse 465
Secant, Log of 19, 513
Secant, Natural 19, 93–94, 172, 513, 515, 556–561
Selectron 378–379, 444, 544
Series, Double 399–400
Series, Summation of 87, 170, 466, 475–476, 520
Sine, Hyperbolic 86–87, 93, 170–171, 205, 336, 463, 465–466, 514
Sine, Hyperbolic Inverse 517–518
Sine, Inverse 85, 87, 463, 465
Sine, Log of 19–20, 85, 87, 355–356, 370, 463–467, 513–514
Sine, Natural 19–20, 85–87, 93–94, 172, 335, 370, 413, 463–467, 485, 513–516, 556–561
Sine Integral 104, 334, 336, 338, 417, 479–482, 514, 526
Sine Integral, Iterated 526
Slide Rule 19, 58, 134–137, 221, 329, 388–389, 446–447, 493, 548
Snedecor's Ratio F 363–364, 466
Spectroscopy 27–29, 255
Spherical Area and Volume 463
Spherical Functions 369
Spherical Harmonics 365
Spherical Triangles 32, 556
Spheroidal Wave Functions 99–101, 524–526
Square Endings 21–22
Square Root 19, 60, 84–85, 87, 98, 171, 302, 340, 370, 463, 466, 494, 513–514
Square Root of Sum of Squares 447
Squares 19–20, 84–85, 87, 171, 370, 412–413, 463, 466, 513–514
Statistics 98–99, 125–126, 133–134, 179–180, 300–303, 359–364, 372–373, 415, 426, 466, 485, 515, 519–521, 524, 546–547
Stirling Numbers 463
Stochastic Methods 546–547
Stokes' Functions 366–367
Struve Functions 66, 103, 340, 365–366
Student's t-Distribution 466, 520, 524
Sum of Powers of Divisors 23, 177
Surveying 493
Symmetric Functions 24, 414

Tangent, Hyperbolic 86–89, 296, 463, 465–467, 514
Tangent, Hyperbolic Inverse 517–518
Tangent, Log of 19–20, 85–87, 355–356, 370, 463–465, 513
Tangent, Natural 19–20, 85–89, 93–94, 146, 172, 296–298, 370, 463, 466–467, 513–515, 556–561
Telephony 98–99
Tetragamma Function 483
Tetrahedral Numbers 316, 423–424
Thermodynamics 511
Theta Functions 89, 104, 229, 235–236, 239–241, 244–245, 257, 334, 369
Toroidal Functions 103, 250
Transcendental Equations 173–175, 178, 414, 469, 495–496, 562
Triangles, Solutions of 556, 561–562
Trigamma Function 483
Trigonometric Functions (special) 19–20, 21, 88, 93–94, 296, 356, 451, 467
Trigonometric Series 368
Trigonometric Sums 472

Universal Automatic Computer (UNIVAC) 133, 341–350, 378, 382, 540

Van der Pol's Equation 108
Versed Cosine 94
Versed Sine 20–21, 94

Waring's Problem 24, 423–424
Wave Equation 106
Weber Functions 103, 365–366
Wedge Functions 105
Weierstrass Function, Elliptic 104, 229, 247–250, 254
Weights and Measures 85, 369, 513
Whirlwind Computer 444–445, 540
Whittaker Functions 104

Zeta Function, Jacobi *see* Jacobi Zeta Function
Zeta Function, Riemann *see* Riemann Zeta Function
Zonal Harmonics 514, 521

Name Index

There are about 15 names where the determination of appropriate initials was beyond our powers.

Abel, N. H. 104
Abraham, M. 481
Abramowitz, M. 477
Abû'l Hasan Ali 226
Adams, E. P. 88, 314, 423, 463
Adams, O. S. 249–250, 263
Adcock, W. A. 221
Adriaen Metius 560
Ageton, A. A. 36, 114–116, 145
Ahlfors, L. V. 546
Aiken, H. H. 1, 2, 4, 54, 58, 102, 130, 161, 164, 185, 188, 210, 311, 327, 367, 377, 382, 439, 446, 491–492, 499, 517, 545
Airey, J. R. 25, 102, 107, 116–118, 143–144, 199, 233, 239, 250, 261, 268, 314, 335–336, 338
Airy, G. B. 337, 339
Aitken, A. C. 300, 399–400, 469
Akhmatov, V. V. 420
Akushskiĭ, I. Ĭa. 60, 221, 299, 314
Aldis, W. S. 549
Alexander, S. N. 128, 380, 438, 545
Alger, P. L. 25
Al-Khowārizmi 227
Allan, F. E. 300
Allen, E. S. 232, 250
Allen, H. F. 382
Alphonse X 563
Alt, F. L. 1, 13, 44, 69, 84, 121, 133, 159, 161, 179, 380, 382, 440, 446
Ambrosio, A. 382
Ambrosio, B. F. 382
Ames Aeronautical Laboratory 199
Anan'ev, I. V. 93, 116
Anderson, E. W. 34
Anderson, R. L. 360
Anderson, Ruth K. 382
Anderson, T. W. 547
Anding, E. 25, 143
Andoyer, M. H. 43, 94, 237, 250, 296–297 426, 515–516, 558
Andrews, E. G. 2, 133, 179, 445, 490
Andrews, E. J. 382
Anger, C. T. 103, 365–366
Anjema, H. 172, 331
Anno, S. 382
Anthonisz, A. 560
Aquino, F. R. de 35–36
Archibald, R. C. 20–21, 25–26, 61–62, 66, 85–87, 89, 92, 94, 102, 105, 117, 142–143, 145–146, 170, 172–173, 180, 186–187, 203, 225, 227, 229–230, 255, 270, 272, 276, 278, 298, 307, 330, 356–358, 366–367, 369, 372, 396, 398, 415–418, 425, 439, 451–452, 463, 467, 475, 482, 493, 495, 498, 514–517, 520–521, 523, 526, 549, 559–561
Archimedes 560
Arenberg, D. L. 479, 540
Arenz, R. F. 382
Arndt, L. 248, 250
Arnold, H. A. 481
Arnold, Phyllis 161
Arnquist, W. N. 382
Aroian, L. 373
Arrow, K. J. 382
Asplund, S. E. 382
Atkinson, E. 256
Avery, H. T. 382
Ayachit, G. R. 360
Azeredo Rodrigues, O. A. de 114, 529

Babbage, Charles 53, 143, 210, 215, 396, 432, 493
Babbage, R. H. 432
Babcock, A. B. 329
Babcock, H. A. 382
Badellino, M. 25, 415

Bailin, L. L. 382
Baily, F. 67
Baldwin, L. U. 382
Baldwin, W. W. 382
Ball, W. W. R. 224
BRL (Ballistic Research Laboratories) 375, 377, 380, 407, 436
Balzer, F. 136
Banachiewicz, T. 13, 16
Banos, A., Jr. 382
Baracket, A. J. 135
Barlow, P. 66–67, 84, 230
Barnes, J. L. 183–184, 382, 545
Barnett, J. 452
Bartky, W. 231, 243, 250
Bartlett, N. R. 60
Bartol Res. Foundation 485
Bateman, H. 58, 141, 229, 239, 241
Bateman, Joan M. 141–142
Baudouin, G. 58
Baugh, A. R. 382
Bauschinger, J. 143, 192, 419
Baxter, Elizabeth P. 383
Beall, G. 356
Beckenbach, E. F. 382–383
Becker, G. F. 143, 200, 223, 225, 371
Bednare, F. J. 383
Beeger, N. G. W. H. 176, 332
Beer, M. 66
Begonich, N. 383, 452
Bell, C. 383
Bell, E. T. 141, 518–519
Bell Telephone Labs. 179, 375–377, 382, 433, 440, 482, 491, 539
Bellman, R. 383
Belzer, J. 544
Beman, W. W. 383
Bems, B. M. 383
Benediht, E. T. 383
Benfield, A. E. 540
Benjamin, K. 135
Bennett, A. A. 520
Bennett, R. 161
Bennett, R. R. 512
Bennett, W. R. 303
Benson, A. I. 383
Benton, A. 383
Berger, A. 26, 473
Bergman, S. 60, 546
Bergmann, S. 247, 250
Bergmark, E. R. 383
Bergson, H. L. 543
Berkeley, E. C. 57, 133, 210–211, 445, 490, 544
Bernoulli, D. 487–488
Bernoulli, Jacques 146, 170, 368, 476, 482, 514
Bernoulli, Joh. 171–172, 559
Berry, C. E. 61, 383
Bers, L. 546
Bertrand, J. L. F. 233, 235–236, 242, 250, 261, 267, 275, 278, 314, 423, 449
Berwick, W. E. H. 336
Bessel, F. W. 25–26, 41, 65–66, 88, 102–103, 105–106, 116–117, 143, 180–181, 183, 185, 187, 196, 199, 203, 229, 253, 300, 305, 315, 333, 335–337, 339–340, 351–352, 361, 365–367, 415–417, 422, 449–450, 466, 470, 474–479, 484, 496, 529, 533, 549–550
Beyer, R. T. 493
Bickley, W. G. 25–26, 333, 337, 339–340
Bieber, V. E., Jr. 383
Bielefeldt, J. A. 383
Bierens de Haan, D. 143, 224, 498
Bigelow, J. H. 378, 440
Bilgeri, M. 560
Billings, J. S. 62
Birge, R. T. 300
Bisch, P. E. 383
Bishop, W. W. 397
Björnbo, A. A. 560
Black, B. J. 135
Blackwell, D. H. 383, 547
Blake, F. G., Jr. 304, 481
Blanch, Gertrude 101, 185, 234, 250, 256, 383, 478, 526, 544
Blewett, J. P. 99
Blinn, W. J. 383
Bliss, C. I. 301
Bloch, R. M. 295, 317, 322, 432
Blythe, J. H. 383
Boaga, G. 528
Boas, R. P., Jr. 183
Bôcher, M. 105, 107
Bode, H. W. 179
Bode, J. E. 137
Bodewig, E. 488
Bodwell, C. A. 383
Boehr, J. W. 383
Boelter, L. M. K. 381, 383
Bohlin, K. 233, 235, 242, 250, 272, 307
Bohnenblust, H. F. 546
Boll, M. 93, 223, 225, 232, 234, 239, 242, 250, 296, 466, 485, 518, 531
Bollay, E. 383
Bollay, W. 383
Bolles, E. E. 383
Bolton, W. W. 383
Boltzmann, L. 350

Bond, H. 224
Bonneau, J. 87
Bonness, J. J. 383
Booth, A. D. 136, 214, 216, 488, 542
Booth, J. 92
Boothroyd, A. R. 540
Borda, J. C. 143–144
Borden, J. R. 383
Bordoni, P. G. 366
Borgnis, F. 106–107
Born, M. 242, 250
Bortkiewicz, L. 180
Bose, P. K. 415
Bose, R. C. 415
Bosmans, H. 559
Bosshardt, R. 172
Bothel, J. R. 446
Bourquard, F. 37, 40, 147
Boussinesq, J. 135
Bouton, C. L. 389
Bouwkamp, C. J. 99, 101, 303–304, 481, 524–526, 531
Bowditch, N. 44, 115, 420
Bower, B. B. 383
Bower, E. C. 383, 419, 425–426
Bowman, F. 41, 267
Boyd, Bettie 179
Boyden, R. E. 383
Bradburn, J. R. 383
Brady, F. H. 383
Brainerd, J. G. 184, 540
Bramble, C. C. 382–383, 446
Branchflower, D. R. 383
Brancker, T. 172
Brandenburg, H. 40, 43, 88, 136
Brandstatter, J. J. 383
Brasey, E. 25
Braudon, C. 383
Brauer, E. 365–366
Brauer, P. 365–366
Braun, E. L. 383
Braun, J. H. 383
Braunmühl, A. 226, 397, 560, 562
Bremiker, C. 136, 467, 561
Bridger, M. 340
Briggs, H. 67, 136, 515–516, 531, 562
Brillouin, L. 212, 431, 435
BAASMTC 25, 90, 92, 107, 117, 180, 187–188, 199, 245, 250, 253, 257, 315, 333–334, 338, 396, 415, 424, 450; *see also* RSMTC, 478, 483
Britten, K. H. V. 216, 488
Brittle, G. N. 383
Broadbent, T. A. A. 550
Brocot, A. 462
Brode, W. R. 381, 383
Bromberg, R. 383
Bronwin, B. 97
Bronzo, J. A. 540
Bross, I. J. 383
Brouwer, D. 237, 251
Brown, B. H. 120
Brown, Bernice 383
Brown, E. W. 143, 237, 251, 259, 361
Brown, F. W. 383
Brown, G. W. 329, 383, 547
Brown, J. C. 224
Brown, R. 543
Brown, R. R. 383
Brown, R. T. 548
Brown & Sharpe 61
Bruhns, C. C. or K. C. 88, 560
Bruk, I. S. 61
Bruno, F. Faà di 24
Bruns, H. 248–249
Bryan, F. E. 383
Buchanan, J. M. 383
Buchholy, W. 383
Buckingham, E. 413, 423
Buckley, H. 390
Büchner, J. P. 171
Buell, E. L. 383
Bürgi, J. 136, 143–144
Bullard, E. C. 417
Buniakowsky, V. J. 249
Burckhardt, J. C. 334, 531
Burdwood, J. 32
Bureau of Census 350
Burgess, G. K. 176
Burington, R. S. 232, 251
Burks, A. W. 50, 56, 128–129, 131, 211, 227, 323–324, 443
Burrau, C. 143
Burton, Patricia 383
Busemann, A. 37, 40
Bush, V. 54, 56, 127
Bussard, R. W. 383
Butter, F. A., Jr. 383
Butterworth, S. 239, 251
Byerly, W. E. 16, 242, 246, 251, 258

Caesar, Julius 514
Cahen, E. 95, 476
Cahn, A. 383, 544
Cajori, F. 224, 391, 397
Caldirola, P. 84
Caldwell, S. H. 61, 136, 438–439, 492, 544
California, Univ. of, Statistical Lab. 372–373
Callandreau, O. 249, 251

Callet, F. 88, 413, 425–426
Cambi, E. 42, 180–181, 186, 199, 231, 234, 251, 304–305, 367
Cameron, R. H. 383
Camp, K. 388
Campbell, D. F. 242, 251
Campbell, G. A. 181–182
Campbell, R. V. D. 57, 133–134, 295, 317, 322, 382–383, 433, 446, 481, 490
Cannon, E. W. 44, 64, 121, 134, 206, 317, 373, 383, 426, 486, 488, 491, 532, 544
Cantor, M. 397, 560
Caples, D. B. 383
Cardwell, W. T., Jr. 383
Carini, W. P. 383
Carle, N. A. 388
Carslaw, H. S. 178
Carter, A. E. 548
Cashen, R. O. *see* Tysser, *Mrs.* R. St. H.
Caspar, M. 560
Cassen, B. 383
Cassini, J. 563
Cassity, C. R. 383
Catalan, E. C. 238
Catlett, W. J. 36
Cauchy, A. 476
Cayley, A. 223, 225, 334–335, 338
Cazzola, E. 84–85, 116
Cecconi, A. 233, 240, 251
Cesareo, O. 127
Cesàro, E. 146
Chamaleix, M. L. 309–310
Chambers, W. & R. 40, 86, 205, 314, 464
Chandler, G. H. 223, 225
Chandler, W. W. 323
Chaplygin, S. A. 522
Chapman, J. 552
Chappell, E. 426
Charlier, C. V. L. 180, 471, 521
Charney, J. G. 102
Chaundy, T. W. 335
Chebyshev, P. L. 17, 64, 97–98, 103, 116, 119–120, 144, 300, 405–407, 471
Chernac, L. 331
Cherry, E. C. 540
Cherry, L. H. 383
Cheydleur, B. F. 544
Chiappinelli, B. A. 383
Christmann, J. 557–558
Christoff, C. A. 383
Christoffel, E. B. 26, 41, 103, 416, 473
Chu, C. 442, 540
Chu, L. J. 101, 526
Church, R. 447
Churchill, R. V. 183
Clark, D., Jr. 258, 383
Clark, K. 328
Clarke, Martha 178
Clausen, T. 368
Clauser, F. H. 383
Cleaver, F. 337
Clerke, A. M. 67
Clifford, W. K. 478
Clippinger, R. F. 133, 207, 446, 544
Coarza, J., Jr. 383
Cochran, W. G. 135, 361, 363
Cohen, A. A. 488
Cohen, H. 498
Cohen, H. G. 540
Cohen, L. 232–233, 238–239, 251, 258–259
Cohn, B. 143
Coleman, B. L. 227
Collar, A. R. 166
Collatz, L. 496, 562
Collie, C. H. 415
Columbia Univ. Press 26
Comfort, E. G. H. 201, 371, 425, 485
Comrie, L. J. 41, 53, 57, 86, 88, 117–118, 143–144, 204–205, 211, 223, 225, 230, 264–265, 267, 276, 314, 336–339, 361, 389, 424, 452, 464–465, 514
Concordia, C. 380, 383
Condon, E. U. 64, 212, 389, 491
Conforto, F. 244, 251, 260
Conn, J. F. C. 121, 407
Coombs, A. W. M. 323
Coombs, J. M. 134, 489
Cooper, E. P. 546
Cooper, R. I. B. 417
Copernicus, N. 394, 553–554, 557, 560
Copsey, E. H. 399–400, 473
Coriolis, G. G. 33, 35
Cornu, A. M. 146, 479
Corrington, M. S. 42, 62, 308, 315
Cossar, J. 183
Cotes, R. 470
Couffignal, P. L. 58, 212, 378, 439
Coulbert, C. D. 383
Coulomb, C. A. de 512
Courant, R. 400, 436, 545
Cox, W. E. 383
Cracknell, A. G. 87
Craig, B. M. 383
Crawford, *see also* Lindsay, J. L.
Crawford, P., Jr. 133, 381, 383
Creak, T. G. 357, 531
Crelle, A. L. 41, 136, 449
Cremona, L. 467, 561
Cresson, E. 63
Criner, H. E. 512

Cross, M. C. 30
Cross, P. C. 27–29
Crout, P. D. 302, 469
Cugle, C. H. 32
Cummings, C. I. 383
Cunningham, A. J. C. 96, 143–144, 333, 335–339, 357, 486, 531
Curry, H. B. 61, 212
Curtis, H. L. 238, 251, 259
Curtiss, J. H. 57, 64, 132, 134, 380–381, 383, 443, 545–546
Curtze, M. 560
Cusa, Nicholas, *see* Nicholas de Cusa.
Cutler, Leola 383

Dahlgren, E. W. 83–84, 382, 433
Dale, J. B. 223, 225, 232, 242, 251, 262, 314, 514, 531
Dancoff, S. M. 383
Danielson, W. D. C. 450–451
Danjon, A. 514
Dantzig, G. B. 133, 382–383
Dantzig, T. 383
Dapron, F. E. 383
Darmstadt Techn. Hochschule 137
Darwin, C. 214
Dase, Z. 143, 334
Daus, P. H. 383
Davidson, Nancy L. 383
Davis, C. F. 384
Davis, H. M. 540
Davis, H. T. 143, 181, 300, 303, 358, 415, 423–424, 450, 463, 473, 483
Davis, P. L. H. 32, 147
Davison, *see* Devison
Davy, H. 66
Day, L. E. 384
Debye, P. 365
Dederick, L. S. 437
Degen, C. F. 335
DeGroot, A. 447
Delambre, J. B. J. 394, 396, 556–557, 559
Delmonte, J. 384
DeMarcus, W. 547
DeMorgan, A. 67, 84, 224, 308, 314, 394–396, 531, 549, 556–557, 559–561
Dench, E. C. 447
Dennis, P. A. 384
DePrima, C. R. 384
Derr, W. A. 389
Descartes, R. 174, 340
Deschodt, J. 474
Dethlefsen, D. G. 384
Dettwiler, H. 136
Deverall, L. I. 384
Devison, B. B. 241, 251
Dewey, C. B. 384
Dibble, E. S. 384
Dickson, L. E. 24, 120, 143, 336, 339
Dienesch, J. 398
Dietzold, R. L. 138
Dijksterhuis, E. J. 224
Dill, D. G. 384
Dillon, J. C. 384
Dimsdale, B. 488, 544
Dinnik, A. N. 25, 365–366, 478
Diophantos, 95, 177, 333
Dirichlet, P. G. L. 60, 221
Dirkes, R. F. 438
Ditkin, V. A. 299, 314
Dixon, A. C. 229, 249–250
Dixon, W. J. 316
Djure, S. 323
Dodson, J. 67, 563
Doetsch, G. 183
Donahue, J. 384
Donnay, J. D. H. 413
Doob, J. L. 547
Doodson, A. T. 335–336
Doolittle, E. 249
Doppler, J. C. 133, 373–374, 446
Dorodnitsyn, A. A. 108
Dorsey, N. E. 238, 251, 258
Dougall, J. 107
Douglass, P. W. 384
Drayton, M. 552
Dreisonstok, J. Y. 420
Dresher, M. 384
Driggers, Mary 384
Dronrijp, Magdalena C. 331
Drucker, D. C. 483
Duarte, F. J. 95
DuBois-Reymond, P. 550
Duncan, D. B. 13, 16
Duncan, W. J. 166
Dunford, J. M. 384
Dunsmore, C. L. 384
Durfee, B. M. 149
Durfee, W. P. 532
Dwight, H. B. 25, 143, 201, 223, 232, 234, 251, 262, 305–306, 314, 423, 482
Dwyer, P. S. 13, 16, 135
Dyer, H. S. 135
Dykstra, C. 381

Eastman, G. 441
Eaton, B. G. 384
Eberlein, W. F. 101
Ebsen, J. 31–32
Echdahl, D. E. 384

Echols, R. L. 547
Eckardt, J. H. 451
Eckert, J. P., Jr. 129–131, 443–444, 491, 540
Eckert, W. J. 60, 135, 145, 161, 206, 217, 327, 355, 432
Eckert-Mauchly Computer Corp. 327–328, 350, 378, 380, 491
Eddington, A. 336
Eddy, R. P. 496
Edlefsen, N. E. 382, 384
Edmonds, C. K. 397
Edmundson, H. P. 384
Eisenhart, C. 64
Ekelof, S. 328
Elbourn, R. D. 326
Elconin, V. 384
Elderton, E. M. 180
Eliot, George (*pseud.*) *see* Evans, Mary A.
Ellis, A. T. 384
Ellis, M. 295, 317, 322
Ellis, W. 296
Emde, F. 25, 41, 62, 88–89, 106–107, 144, 176, 231–232, 234–236, 238–242, 244, 247, 249, 252–255, 263, 267–270, 275, 296, 305, 307, 314–315, oppo. 333, 364–365, 371, 397–398, 416–417, 423, 450, 452, 479, 482, 517
Emden, R. 336, 339
Emmons, H. W. 36, 40, 198, 437
Emslie, A. G. 540
Encke, J. F. 419, 470
Engel, E. 146
Enger, M. L. 384
Englund, C. R. 451
Epstein, A. L. 201
Erdélyi, A. 57–58, 141, 183
Ericsson, L. M. 323
Ericsson, S. 492
Erlang, A. K. 98–99
Euclid 66, 357, 528
Euler, L. 95, 146, 184, 189, 255, 340, 358, 452, 467–468, 475, 479–480, 482, 495, 514, 526, 531, 551
Evans, Mary A. (G. Eliot) 551–552
Everett, J. D. 187, 339, 366, 418–419, 470, 472, 477–478
Everett, R. R. 384

Fabry, C. 176
Fadle, J. 414
Fairbanks, A. F. 384
Fairthorne, R. A. 136, 400
Fano, U. 547
Farey, J. 333, 339
Farley, R. 66
Faucher, C. 425
Fauquembergue, E. 496
Fawks, G. R., Jr. 384
Feferman, S. 384
Feldheim, E. 167
Felkel, A. 171, 222
Feller, W. 98–99, 179, 301–302, 317, 358–360, 474, 520, 524, 531, 547
Fender, F. G. 384
Fenn, G. S. 384
Ferber, B. 384
Ferguson, D. F. 18–19, 334, 340
Fermat, P. 314, 451, 493, 496, 518
Ferrari, C. 356
Ferrari, K. G. 192–195, 199
Ferrier, A. 95, 451, 497
Feshbach, H. 107
Fibonacci (Leonardo of Pisa) 96, 119, 299
Filipowski, Z. H. or H. E. 66–67
Filon, L. N. G. 335, 338
Financial Publ. Co. 30, 527
Fincke, T. 397
Findley, W. G. 135
Finney, D. J. 359–360, 364
Fischer, W. L. F. 467, 561
Fisher, R. A. 300, 333, 336–337, 359–360, 362–363, 371, 426, 515
Fix, Evelyn 373
Flanagan, J. C. 135
Fletcher, A. 29, 31, 61, 92, 108, 119, 188, 203, 223, 229–230, 232–233, 237, 240, 252, 276–277, 305–307, 314, 331, 365, 371, 449–451, 477, 495, 498, 521, 531, 551
FMR 31, 61, 108, 119, 203, 223, 240, 276–277, 314, 371, 495, 531, 551
Flood, M. M. 88, 384
Follin, J. W., Jr. 384
Fontoura da Costa, A. 114–116, 145
Forbes, S. H. 485
Forrester, J. W. 382, 384, 435
Forsdyke, A. G. 239, 252, 256
Forster, H. K. 384
Forsyth, A. R. 335, 338, 539
Forsythe, G. E. 384, 547
Forsythe, W. E. 176
Forti, A. 201, 223, 225
Foster, R. M. 181
Fouquet, W. 239, 252
Fourier, J. B. J. 16, 21, 61, 82, 104, 136, 181–182, 241, 247, 251, 400, 413, 434, 471, 475, 544
Fout, A. A. 384
Fowle, F. E. 176, 232–233, 238, 252–253

Fox, E. N. 246, 252
Fox, L. 132, 462
France, Inst. Géogr. Nat. 355, 371
France, Service Géogr. de l'Armée 355
France, Service Technique Aéronautique 309–310, 314, 371
Frankel, S. P. 384
Franks, J. R. 384
Fraser, C. H. 384
Frazer, H. 399–400, 473
Frazer, R. A. 166
Frazier, T. V. 384
Frederic, O. C. E. 384
Frederick II 397
Frederick IV 390–391, 394, 554–557
Freeman, J. G. 29, 243, 252
Frehafer, Mabel K. 176
Fresnel, A. J. 104, 146, 336, 417, 467, 479, 550
Freund, J. 384
Frick, R. H. 384
Fricke, R. 232, 235, 242, 252
Fridén, C. 375
Friedman, B. 382, 384
Frisby, E. R. 384
Frobenius, G. 533
Fröberg, C. E. 492
Frost, A. E. 438
Fruitman, F. 384
Fry, B. L. 384
Fry, T. C. 99, 138, 306, 521
Frye, W. E. 384
Fuchs, H. O. 384
Fürth, R. 61
Fuess, Barbara F. 384
Fuller, H. W. 432, 438

Gabba, L. 20, 40
Gabler, R. T. 384
Gadd, J. O., Jr. 481
Galton, F. 337
Galvan, Marjorie 384
Gamble, *Miss* N. 418, 423
Garabedian, P. R. 546
Gardiner, W. 88
Gardner, M. F. 183–184
Garfinkel, B. 377
Garreau, G. A. 340
Garrick, I. E. 522, 545
Gass, 396
Gauss, A. F. G. T. 86
Gauss, C. F. 63, 86, 97, 104, 161, 193, 201, 221, 233, 245, 247, 252, 254, 262, 314, 350, 416, 419, 470–473, 493, 515, 528, 531, 551
Gauss, F. G. 85–86, 136
Gavrilko, G. I. 222
Gegenbauer, L. 103
Gellibrand, H. 515–516
George Eliot (*pseud*), *see* Evans, Mary A.
George of Trebizond 562
Gerlough, D. L. 384
Germany, Deutsche Seewarte 31, 109, 116, 138–139
Germany, Reichsluftfahrtministerium 32
Germond, H. H. 384
Gernerth, A. 395
Ghizzetti, A. 415
Gibbs, J. W. 434, 543
Gibson, S. O. 384
Gifford, Emma 296–298, 314, 337, 339
Gilbert, P. 146
Gilbert, R. E. 389
Gilmore, F. R. 384
Ginsburg, J. 66
Girard, A. 451, 561–562
Girshick, M. A. 384, 547
Glaisher, J. 143, 172, 334, 338
Glaisher, J. W. L. 89–90, 92, 108, 120–121, 143–144, 167, 170, 172, 201, 226, 234–236, 241, 244–247, 252, 257, 262–263, 314, 334–339, 358, 372, 396, 424, 463, 531, 557, 559
Glazenap, S. P. 62, 232, 235, 242, 252, 268, 272, 307, 423
Globe, S. 58–59
Gloden, A. 21, 87, 96, 118–120, 298, 357, 486
Gobbin, H. H. 85, 136
Godfrey, G. H. 482
Gold, E. 108
Goldberg, E. A. 329, 447
Goldman, S. 416, 423
Goldschmidt, G. H. 412
Goldstein, L. 302
Goldstein, S. 106–107
Goldstine, Adele K. 323
Goldstine, H. H. 50, 54, 56, 128, 131, 166, 216, 323–324, 382, 384, 462, 541
Goodpasture, J. J. 384
Goodwin, E. T. 333–334, 337, 340, 483
Goodwin, G. 67
Gorman, J. R. 384
Gortzel, G. H. 547
Gosset, W. S. (*Student*) 363
Gossot, H. C. F. F. 247, 252–253, 263, 314
Gourier, G. 473
Gourrich, G. E. 384, 488
Gradshtein, I. S. 299
Graeffe, C. H. 487–488, 494
Graham, R. B. 384

Graindorge, J. 253, 259
Gram, J. P. 358, 471, 521
Grandi, L. L. 384
Grassman, H. G. 449
Gravelaar, N. L. W. A. 391, 396
Gray, A. 117, 232, 238, 253, 423, 499–450
Gray, G. J. 67
Gray, H. J. 184
Gray, Marion C. 306, 482
Gray, P. 66–67
Gray, T. 252–253
Great Britain, Admiralty Computing Service 57, 103, 241, 253, 367
Great Britain, Ministry of Supply 136
Great Britain, Nautical Almanac Office 34, 94, 110, 204, 310
Great Britain, Post Office Engin. Dept. 323
Green, J. 384
Green, J. W. 546
Green, L. 178
Greenberg, Celia E. 384
Greenberg, H. 384
Greenfield, M. A. 384
Greenhill, A. G. 89–91, 223, 225, 233, 242, 244–247, 249, 252–253, 263, 268, 314, 335
Greenwood, J. A. 36, 180, 426
Greenwood, R. E. 384, 416
Gregory, J. 224, 331, 358, 470
Griess, J. 253
Griest, R. H. 384
Griffin, Amy E. 384
Griffin, F. L. 242, 253
Griffith, F. M. 384
Griggs, D. T. 384
Grobecker, A. J. 384
Gröbner, W. 482
Gronau, J. F. W. 222
Grosch, H. R. J. 135
Gross, G. L. 474
Gross, O. A. 384
Grover, F. W. 232–235, 238–239, 253, 259, 521
Grunert, J. A. 449
Grynaeus, S. 562
Gudermann, C. 40–41, 201, 223, 225, 244
Günther, S. 560
Guldhammer, H. A. 369, 371
Gumbel, E. J. 426
Gunning, W. F. 384
Gunter, E. 136
Gupta, H. 22–23, 298, 334, 340
Gutshall, W. D. 384
Gwyther, C. E. 334, 337, 339–340, 450
Gyldén, H. 237, 253
Habenstreit, W. B. 384
Hacker, S. G. 492
Hadcock, A. G. 247
Hadlock, C. K. 384
Haefele, F. 560
Haeff, A. V. 286, 317, 379
Hage, J. 29, 40
Hagner, F. H. 59
Hahn, W. C. 136
Haight, G. S. 552
Hainer, R. M. 27–29
Hak, J. 238–239, 253
Hales, J. V. 384
Hall, J. R. 384
Hall, W. 223, 225
Hallén, E. 303, 481–482, 526, 531
Halley, E. 224–225, 553, 563
Hamann, C. 136–137
Hamburger, G. E. 413
Hameister, E. 184
Hamilton, A. 489
Hammad, A. 108
Hammer, H. C. 308, 314
Hammer, P. 547
Hanafi, A. R. 24, 40, 414, 423
Hancock, H. 232, 242, 253, 263, 314
Handson, R. 392–393, 397, 499
Hankel, H. 104, 299
Hansen, P. A. 418
Harbert, E. 86, 136
Harder, E. L. 512
Hardgrave, E. J., Jr. 384
Hardy, A. C. 447
Hardy, G. H. 314, 399–400
Hare, G. 384
Harley, R. 335, 338
Harr, J. A. 186, 188, 367, 475
Harris, T. E. 384, 547
Harrison, C. W., Jr. 481–482
Harrison, F. 423, 485
Harrison, J. O., Jr. 407, 426, 437
Hart, H. C. 138
Hart, W. L. 412
Hartig, H. E. 174–175
Hartley, H. O. 337
Hartree, D. R. 56–57, 64, 127, 131, 208, 325, 381–382, 384, 516, 521, 542
Harvard Univ., Computation Lab. 41, 55–56, 69, 79, 83, 102, 117, 130, 161, 166, 181, 185, 311, 314, 367, 381, 416, 432–433, 436, 438–439, 446, 450, 474, 479, 499, 517, 540, 545
Harwood, R. H. 384
Haseltine, W. R. 384
Hasted, J. B. 415

Hastings, R. E. 384
Haues, R. M. 385
Hausman, Lillian F. 160–161
Hay, H. G. 418, 423
Hayashi, K. 25, 40, 42–43, 102, 181, 186, 223, 225, 230, 232–233, 235–236, 242, 253, 263–265, 276, 278–279, 296, 314, 367, 475, 518
Hayden, C. 221
Hayford, J. F. 529
Hazen, J. W. 385
Heaviside, O. 182
Hedeman, W. R., Jr. 390
Hegbar, H. R. 385
Heimlich, I. R. 385
Heins, A. E. 476
Helmer, O. 385
Helmholtz, H. 481, 545
Henderson, J. 88, 333, 336–337, 339, 394, 467
Henrion, D. 394
Herbig, Delia M. 385
Herget, P. 60, 418–419, 423
Herman, H. L. 385
Hermite, C. 26, 103, 167, 180, 234, 253, 338, 365, 416, 471, 473
Herold, K. 255
Herrick, Marjorie 161
Herrick, S. 59, 385
Herrmann, 515–516
Herwald, S. W. 512
Hestenes, M. R. 385
Hetzer, E. M. R. 192, 195
Heuman, C. 27–29, 233–234, 239, 243, 254, 266, 270–272, 307, 314
Heun, K. 132, 437, 443
Hickerson, T. F. 115, 452
Hicks, B. L. 133, 208
Hicks, W. M. 335
Higgins, A. L. 146
Higgins, T. J. 382, 385
Hilbert, D. 104, 399–400, 473
Hildebrandt, T. W. 440
Hill, B. I. 385
Hill, G. W. 240–241, 249, 254, 305
Hill, M. J. M. 335, 338
Hillam, K. L. 385
Hillman, A. P. 352, 414
Hilsenrath, J. 58–59
Himes, H. W. 385
Himes, P. J. 385
Hindenburg, K. F. 171
Hinkley, E. 172
Hipler, F. 559
Hippisley, R. L. 88–91, 232, 234, 236, 245–246, 253–254, 263, 266, 314, 336
Hirayama, A. 532
Hirsch, C. J. 491
Hirsch, M. 482
Hoagland, A. S. 385
Hoberg, G. G. 489
Hobson, E. W. 16, 223, 225, 335
Hochwald, W. 385
Hodgman, C. D. 232–233, 254
Hoehne, G. G. 34, 529
Hoëne-Wroński, J. M. 143, 539
Hoffleit, Dorrit 133, 377, 446
Hoffman, W. C. 385
Hofreiter, N. 482
Hohenemser, K. 396
Holberton, J. V. 133, 208
Hollerith, Franciska B. 62
Hollerith, G. 62
Hollerith, H. 54, 62–63, 135, 215, 399
Holliday, F. 463, 551
Holtappel, H. W. 485
Holzer, R. E. 385
Hook, J. F. 385
Hooks, I. J. 135
Hopkins, R. 385
Hoppe, E. R. E. 143
Hopper, Grace M. 210, 377
Horblit, H. D. 392, 397, 499
Horenstein, W. 234, 254, 256, 522
Horgen, Ruth B. 385
Horner, W. G. 486
Horowitz, J. 385
Horton, C. W. 66
Horton, R. E. 385
Hossner, A. 255
Hoüel, J. 88, 91–92, 223, 225, 233–236, 241–242, 244, 254
Householder, A. S. 385, 544, 547
Houseman, E. E. 360
Howland, R. C. J. 472
Hoyt, F. C. 547
Huckel, Vera 522, 531
Hülsse, J. A. 467
Hütte, 232, 242, 254
Huey, R. B. 297–298
Humbert, P. 183
Hunrath, K. 560
Hunt, F. J. 528
Huntington, A. 385
Huntington, E. V. 223
Huntington, H. B. 540
Hurd, C. C. 385
Hurty, W. C. 385
Huskey, H. D. 64, 215, 326, 328, 382, 385, 462, 516, 544–545
Huskey, *Mrs.* H. D. 385

Hutchcraft, H. W. 385
Hutchings, C. H. 33–34, 36
Hutchinson, C. A. 385
Hutner, R. A. 101, 526
Hutton, C. 143, 224, 559, 562
Huygens, C. 144, 562

Ibn al-Zarqāla 563
Idelson, N. 233, 254
Ince, E. L. 336, 339, 531, 539
Inman, J. 201, 223
Innes, R. T. A. 235, 237, 249, 254, 258, 266–267, 275, 314
Inst. Adv. Study, Princeton 440
International Business Machines Corp. (IBM) 2, 27, 54, 60, 63, 105, 133, 135, 145, 149–150, 155, 157, 185, 206–208, 215–217, 325–328, 330, 352–353, 367, 375–377, 381–382, 426, 432–433, 491, 493, 521, 540, 544
Irwin, J. B. 114, 189–191, 385, 440
Irwin, J. O. 333, 336–337, 339
Isaacs, R. 385
Isherwood, J. G. 550
Italy, Consiglio Nazionale d. Ricerche 356
Italy, Istituto Geografico Militare 528
Italy, Istituto Idrogr. d. r. Marina 419, 513

Jackman, Marian C. 301
Jackson, J. B. 132
Jackson, W. A. 499
Jacobi, C. G. J. 89–91, 103–104, 143, 229, 234–236, 241, 243–244, 246–247, 254, 256–257, 350, 449, 482
Jacobs, C. N. 385
Jacobs, E. H. 385
Jacobsthal, E. 372
Jacq, M. J. 309
Jäderin, E. 300
Jaeger, J. C. 178, 417
Jäger, P. 171
Jahnke, P. R. E. 25, 41, 62, 144, 176, 231–232, 234–236, 238–240, 242, 244, 247, 249, 252–254, 263, 267, 275, 296, 305, 314–315, 364, 371, 397, 416–417, 423, 450, 452, 479, 482, 517
James, H. M. 188, 447
Janssen, E. 385
Japan, Hydrogr. Dept. (Office) 110, 195–196, 311, 361
Jeffrey, G. A. 136
Jeffreys, H. 107, 337, 519
Jelínek, L. 233, 255
Joachim, G. *see* Rheticus
Jørgensen, N. R. 62, 180, 521
Joffe, S. A. 20, 93, 178, 299, 369, 564
Johnsen, Madeline 373
Johnson, D. C. 30
Johnson, E. F. 385
Johnson, L. 134
Johnson, P. A. 385
Johnson, W. W. 298
Johnston, H. L. 544
Johnston, S. 340, 521
Joncourt, É. de 171
Jones, C. W. 121, 333, 337, 340, 407
Jones, H. S. 95
Jones, Rebecca 161
Jordan, C. 300, 463, 468
Jordan, W. 136, 298, 356
Josefowitz, S. 389
Joubert, J. 25, 238, 255–256
Juley, J. 127

Kac, M. 547
Kacmarz, S. 407
Kästner, A. G. 396, 559
Kahn, H. 385, 547
Kaiser, H. R. 385
Kalähne, A. 305
Kalin, T. 545
Kallmann, Hildegard K. 385
Kamke, E. 137
Kao, S.-K. 385
Kaplan, C. 522
Kaplan, E. L. 231–233, 242, 255, 264–265, 268, 270–272, 306–307, 314
Kapteyn, J. C. 103
Karîâkin, N. I. 357, 371
Karlin, S. 385
Kármán, T. 259
Karpelesz, K. 67
Kaster, H. 59
Katz, A. 299, 314, 519
Kay, A. F. 385
Keller, G. 161
Kelley, T. L. 98, 301, 423, 485
Kelvin, W. T., *Lord* 25, 334, 338, 340
Kemmer, P. H. 385
Kendall, J. T. 144
Kennedy, E. C. 385
Kennedy, G. 335
Kennelly, A. E. 174–175, 225, 516
Kenney, J. F. 13, 16
Kepler, J. 143–144, 252, 553, 560
Kerawala, S. M. 24, 40, 300, 414, 423
Kerridge, S. 365–366
Kessman, M. 385
Ketchum, P. W. 60, 221
Keye, W. R. 488

Khanov, B. 299
Khristianovich, S. A. 109, 251
Kiely, E. R. 493
Kiepert, L. 232, 242, 255
Kilburn, T. 445
Kimball, R. B. 385
Kind-Schaad, G. 60
King, G. W. 27–29, 60, 188, 243, 255, 355, 545, 547
King, L. V. 230, 481
King, R. I. 304, 385
King, R. W. P. 481–482
Kircher, E. A. 30
Kirchhoff, G. R. 505
Kirkham, W. J. 423, 450
Kirschbaum, H. S. 512
Kirschner, 449
Kitover, K. A. 368
Klemperer, W. B. 385
Kline, M. 305–306
Knauss, H. P. 548
Knott, C. G. 224, 397
Knutson, R. G. 385
Kober, H. 103, 368
Koehler, J. S. 212
Kohlschütter, A. 116
Kondo, T. 180
Koons, Florence 431
Kopal, Z. 37, 60, 112–114, 190–191, 194–195, 197, 473
Kopper, J. M. 512
Korn, G. A. 441
Kornei, O. 438
Koshliâkov, N. S. 26, 41–42
Kourganoff, V. 307
Kraïtchik, M. B. 143, 177, 199, 357, 372, 531
Krall, G. 356
Krasnushkin, P. 107
Kratzer, A. 137
Kreith, F. 385
Krieger, F. J. 385
Kron, G. 222
Krueger, C. N. A. 450
Krüger, J. G. 171
Krüger, J. H. L. 86
Kruse, K. H. W. 144
Krylov, A. 127
Kuehni, H. P. 385
Kulik, J. P. 22, 222
Kulikov, D. K. 21
Kummer, E. E. 104, 477, 523
Kurtz, A. K. 135
Kusner, J. H. 385
Kutta, M. W. 443
Kuznetsev, P. I. 186
Lacroix, A. 84
Laderman, J. 16
Lafferty, R. E. 135
Lagerstrom, P. 385
Lagrange, J. L. 63, 82, 98, 302, 325, 358, 401, 414, 464, 466, 469, 472, 487, 524
Laguerre, É. N. 26, 41, 103–104, 167, 365, 532
Lahiri, D. B. 23, 177
Laitone, E. V. 385
Lake, C. D. 149
LaLande, J. J. F. 143, 391, 396, 557, 559
Lambert, J. H. 145, 170–172, 199, 222–223, 225, 250, 419, 526
Lamé, G. 101, 104, 524
Lanczos, C. 17, 120, 385, 499
Landau, E. 550
Landau, H. G. 208
Landen, J. 230–231, 238, 241, 269
Lane, W. 186, 199
Lange, N. A. 251
Langer, R. E. 187, 477
Laplace, P. S. 61, 104, 106, 178–179, 182–184, 222, 229, 237, 239, 252, 299, 341–345, 350, 373, 418, 470, 476, 515, 523
Lapushkin, 420
Lapworth, N. 385
Larkin, J. J. 385
Larsen, H. D. 466
Láska, W. 232, 234–235, 242, 255, 482
Latter, A. L. 385
Laughton, J. K. 224
Laurent, T. 492
Lawler, Jane M. 385
Lax, M. 107
Lebell, D. 385
Lebesgue, H. 543, 550
LeCaine, J. 526
Lecolazet, R. 369
LeCorbeiller, P. 385
Lee, L. K. 385
Lefschetz, S. 382, 385
Legendre, A. M. 16–17, 27, 64, 89, 99, 103, 105, 143, 167, 186, 194, 223, 231–233, 235, 237, 239, 241–244, 246, 248, 251–252, 255–258, 261–262, 266, 268–272, 274–275, 278, 280–281, 304, 306–307, 314, 331, 334, 337, 339, 365, 369, 470–472, 484, 525, 533
Lehmer, D. H. 21–24, 61, 88, 95–97, 120, 131, 143, 176–177, 206, 227, 298–299, 308, 330, 337, 339, 357–358, 368, 372, 382, 385, 414, 417, 451, 468, 473, 486, 494, 496–497, 518–519, 531–532, 551
Lehmer, *Mrs.* D. H. 564

Lehmer, D. N. 143–144, 146, 222, 334, 358, 452, 531
Leïbin, A. 222
Leibniz, G. W. 136–137, 493
Leiner, A. L. 326, 541
Leipper, D. F. 385
Leitner, F. 101
Leon, H. I. 385
Leontief, W. W. 161, 166, 436
Leupin, E. 136
Leuschner, A. O. 418
Levallois, J. J. 87
Leverrier, U. J. J. 237, 256
Levin, Doris 479
Levin, J. H. 133, 209
Levine, H. 524
Levy, H. 239, 252, 256
Lévy, L. 230, 232, 235, 242, 256, 263, 267, 274–275, 278, 314, 449
Lewis, S. N. 385
Licklider, J. C. R. 385
Liebmann, H. 343–344, 350
Liechtenstein, P. 562
Lienard, R. 358
Liesegang, F. P. 203
Ligowski, J. O. W. 201, 232, 242, 256
Lilley, S. 213
Lindsay, J. L. 26th *Earl of* Crawford 331, 558
Lindsay, R. B. 484, 496
Lindvall, F. C. 512
Linstone, H. A. 385
Lion, A. 379
Lipkis, R. 385
Littlewood, J. E. 399–400
Lĭŭsternik, L. A. 299, 314
Livingston, H. M. 350, 373
Locanthi, B. N. 512
Lodge, A. 25, 143, 186, 335–339, 514
Loe, Alice G. 179
Lo-Ho 447
Lohse, W. O. 143
Lommel, E. C. J. 103, 143, 186, 336, 550
Long, M. V. 385
Loranthe, B. 385
Lorell, J. 385
Lorentz, H. A. 412
Lorne, J. 385
Loss, H. 385
Lotkin, M. 133, 210
Love, A. E. H. 106, 335
Lovett, D. B. 385
Lowan, A. N. 64, 107–108, 143, 173, 187, 203, 234, 250, 254, 256, 315, 414, oppo. 453, 466, 494–495, 522
Loxhay, 235, 280
Lubkin, S. 50, 323, 327, 380, 431, 445, 541–542, 545
Lucas, É. 142–143, 496, 518
Lucas, F. 328–329
Ludlam, A. E. 25
Ludolf van Ceulen 562
Lukacs, E. 58
Lukoff, H. 491
Luth, P. A., Jr., 385
Luther, M. 552
Luxenberg, H. 385

Macaulax, M. 385
McCann, G. D. 382, 386, 511–512, 532
McCarthy, J. 386
McClellan, J. P. 386
McClintock, F. A. 386
Maccoll, J. W. 37–38, 40
McCool, W. A. 542
McCormick, C. W. B. 386
Macdonald, H. M. 335
McEuen, A. H. 386
McEwen, G. F. 386
Macfarlane, G. G. 418, 475
McFarlane, M. D. 386
MacGregor, W. 242, 256
Mach, E. 36–39, 198–199, 206, 522
MacInnes, Janet 330
McKay, A. T. 240, 256
MacKay, D. M. 512
McLachlan, N. W. 183, 185
MacLane, G. R. 188
MacLean, M. 25
Macleish, K. G. 440–441, 452
McMahon, J. 305
MacMahon, P. A. 335
McMillan, B. 382, 386
McNamee, J. 246, 252
MacNeal, R. H. 385, 512
MacNee, A. B. 491
MacNeille, H. M. 385
McNulty, W. P. 386
McPherson, D. E., Jr. 386
McPherson, J. L. 121, 133–134, 380, 446
McQuitty, J. V. 135
MacRobert, T. M. 16, 117, 449–450
Magnus, W. 103, 116, 368, 522, 531
Mahler, K. 23
Mahler, Mary L. 524
Maier, O. C. 381, 385
Mainland, D. 362
Maker, F. L. 385
Mallock, R. R. M. 137
Malmberg, G. 492

Malone, Helen 407, 426
Mandelbrot, B. B. 385
Manley, R. G. 134
Mansell, W. E. 339
Marchant Calc. Machine Co. 375
Marchisio, P. 88, 93
Marcum, J. I. 521
Mark, R. S. 385
Markov, A. A. 97, 143
Marsh, H. W., Jr. 385
Marshall, B. O., Jr. 491
Martelli, G. F. 197
Marth, A. 419
Martin, A. 143
Martin, F. C. 386
Martin, F. T. 386
Martin, W. T. 546
Masal, H. 237, 256
Mascart, É. É. N. 25, 238, 255–256
Maseres, F. 172, 224
MIT, 37, 40, 107, 175, 188, 197, 447
Mathews, G. B. 117, 335, 423, 449–450
Mathieu, E. L. 101, 103–104, 184–185, 366, 525
Mauchly, J. W. 57, 129, 133–134, 380, 382, 386, 437, 442, 444, 491–492, 540
Mauchly, Kathleen R. 386
Maxfield, J. P. 386
Maximilian II 554, 560
Maxwell, J. C. 238, 256, 481
Mayer, Maria 547
Mayo, L. R. 61
Meissel, E. D. F. 25, 102, 117, 181, 186, 235, 256, 261, 263, 267, 275–278, 314, 333, 339, oppo. 399, 416, 423, 449–450, 549
Meixner, J. 524–525, 531
Melahn, W. S. 386
Melanchthon, P. 552–553
Mellin, R. H. 104, 475–476
Mengel, A. S. 386
Mercator, N. 44, 110, 223–224, 530
Mercner, R. O. 138
Merfield, C. J. 249, 256, 269, 272, 307, 314
Meriam, J. L. 539
Merrick, W. D. 386
Merrifield, C. W. 226
Merrill, J. E. 193–194
Merrington, Maxine 360
Merritt, H. E. 462
Mersenne, M. 142, 398
Mersman, W. A. 386
Merwin, R. 184
Mesick, B. S. 386
Metius, *see* Adriaen
Metropolis, N. 386, 547
Mettler, R. F. 386
Meyer zur Capellen, W. 137
Michal, A. D. 386
Michell, J. H. 25, 62
Michelson, S. 227
Middleton, D. 482
Mie, G. 483–484
Mikami, Y. 560
Mikhlin, S. G. 251
Miles, J. W. 386
Miller, A. 386
Miller, F. G. 166, 437
Miller, J. C. P. 31, 41, 61, 108, 118–119, 121, 132, 143, 199, 203, 223, 229–230, 237, 240–241, 263, 276–277, 305, 314–315, 333, 336–340, 355, 371, 400, 407, 426, 450, 495, 522, 531, 551
Miller, J. J. 416
Miller, J. M. 238–239, 256, 258
Milne, W. E. 108, 468, 547–548
Milne-Thomson, L. M. 86, 223, 225, 232–233, 235, 244, 246–247, 256, 264, 276, 278–279, 333, 338, 468
Minding, F. 482
Minkowski, H. 96
Minvielle, L. 386
Mises, R. 545
Mitchell, D. V. 19
Mitchell, F. W. 386
Mitchell, H. F., Jr. 166, 206, 462
Mitchell, K. 477
Mitchell, W. D. 386
Mitrinovich, D. S. 463
Mittelman, Beatrice S. 414
Molina, E. C. D. 301
Molk, J. 234, 247, 257, 260
Mongan, C. E., Jr. 386
Monroe Calc. Machine Co. 375
Monteil, C. 514
Montessus de Ballore, R. de 232, 235, 242, 257
Mood, A. M. 386
Mooers, C. N. 444
Moon, P. 175–176
Moore, B. L. 435
Moore, E. F. 488
Moore, E. H. 389
Moore, J. 233, 257, 276, 314
Moore, J. R. 386
Moore Sch. Electr. Engin., Philadelphia 379, 427, 433, 440, 442, 489, 540
Moretti, G. 116
Morgan, S. P., Jr. 105, 107
Morgan, W. D. 558–559, 562–563
Morikawa, G. K. 386

Morrill, C. D. 386
Morris, A. J. 386
Morrisson, R. 386
Morse, P. M. 25, 101, 107, 526
Morton, P. L. 382, 386
Mose, H. E. 20
Moseley, H. 232, 257, 331
Mosier, C. I. 135
Moskovitz, D. 232, 242, 259, 278, 314
Mossotti, O. F. 225
Mowbray, A. C. 386
Moxon, J. 224
Müller, A. 553, 559–560
Müller, O. 20, 40
Müller, W. C. 298
Muller, M. 386
Munroe, H. D. 386
Murnaghan, F. D. 141–142
Murray, F. J. 138, 330
Myers, J. 386
Mynall, D. J. 512

Nachod, C. P. 447
Nadel, A. 386
Nagaoka, H. 231–232, 234–239, 247, 257, 259, 276, 314
Naiman, J. M. 386
Nair, K. R. 363–364
Napier, J. 67, 136, 143, 224–225, 372, 397, 514, 552, 563
Naske, C. 482
NAML (Nat. Appl. Math. Labs.) 64–65, 445
National Bureau of Standards (NBS) 63–64, 238–239, 253, 257, 259, 286, 326, 350, 431, 438–439, 443, 488, 490, 496, 515–516, 519, 522–523, 541, 544–545, 547
NBS Computation Lab. (NBSCL) 25–26, 35, 40, 42–43, 65, 88, 101, 107–108, 116, 120, 169, 173, 176, 180, 187, 201, 203, 205, 227, 231–235, 239, 246, 257, 296, 315–317, 352, 371, 407, 414, 436, 472, 477, 483, 485, 494, 515–516, 523, 545
NBSINA 381, 425, 478, 485, 544–547
NBS Machine Development Lab. (NBSMDL) 44, 53–54, 56–57, 121, 128, 206, 210–213, 215, 317, 327–328, 373, 378–380, 426, 432, 440–442, 445, 486, 541–542
NBSMTC 481
Nat. Defense Res. Committee (NDRC) 127, 373
Navier, C. L. M. H. 37, 437
Nayler, J. 25
Nehari, Z. 546
Neiburger, M. 386
Neishuler, L. [illegible]a. 299
Nelson, E. 386
Nelson, L. 386, 547
Neumann, C. G. 336
Neumann, J. 2, 50, 54, 56, 58, 128, 133, 166, 216, 324, 381, 386, 439, 446, 462, 541, 546–547
Neville, E. H. 333, 336, 338–340, 469
Newcomb, S. 234, 237, 257
Newman, A. B. 178, 240, 257
Newman, F. W. 201, 231–232, 240–241, 247, 257, 368, 451, 477, 485, 549, 551–552
Newman, M. H. A. 214, 542
Newton, I. 27, 100, 132, 351, 358, 414, 469–470, 472, 486, 543, 553
Newton, R. R. 474
NYWPA, 521
Neyman, J. 373, 547
Nicholas de Cusa 562
Nichols, N. B. 188
Nicholson, J. W. 335–336, 338, 367
Nielsen, K. L. 302
Nielsen, N. 143, 481
Niepmann, H. W. 386
Niepmann, K. 386
Nilson, E. N. 386
Niven, W. D. 256
Nogrady, H. A. 178
Nolde, G. V. 386
Norén, G. 237, 258
Norwood, R. 224
Nowacki, *Miss* H. 203
Numerov, B. 86
Nye, G. 386
Nyquist, H. 179
Nyström, E. J. 244, 258

Oberhettinger, F. 103, 116, 368, 522, 531
Oberreit, 171
O'Brien, P. F. 386
Ocagne, M. d' 136
Odhner, W. T. 136–137
Odle, J. W. 386
Örley, L. 452
Ogura, S. 420
Okamatu, M. 176
Okaya, T. 203
Oldfield, B. G. 386
Olds, E. B. 135
Olds, R. H. 386
Olmsted, E. G. 386
Olson, F. C. W. 240, 258–259
O'Malley, C. A. 386
O'Neal, R. D. 386, 438, 441, 444, 452
O'Neill, R. R. 386

Oppolzer, E. 396
Osbon, W. O. 512
Osborn, J. A. 242, 258
Osborn, P. 386
Oseen, C. W. 481
Osgood, W. F. 258
Ostrogradskiĭ, M. B. 145, 531
Ostrowski, A. 545
Othmer, D. F. 389
Otho, L. V. 394–395, 554–562
Oughtred, W. 136, 372, 398

Packh, D. C. de 222
Padé, H. 260
Page, C. H. 128, 444, 489
Page, L. 101
Pagel, F. L. 530
Paine, A. C. 386
Paine, W. O. 386
Palm, C. C. R. A. 98, 492
Palmer, R. 491–492
Pankhurst, R. C. 121, 407
Panse, V. G. 360
Pareto, V. 300
Parker, J. B. 31–32, 34, 110, 141
Parkin, T. R. 386
Parks, R. J. 386
Pascal, B. 136, 493
Patterson, G. W. 386, 489
Pauling, L. 60
Peake, H. J. 308
Pearson, E. S. 336
Pearson, K. 67, 89, 143, 180, 241, 255, 258, 268, 270–272, 306–307, 335, 338, 372–373, 485
Peebles, G. H. 386
Peirce, B. O. 143, 232, 242, 251, 258, 482
Peirce, C. 386
Pell, J. 172, 335, 338
Pemberton, J. C. 61, 386
Pendergrass, J. T. 326
Penteado, F. 114–116, 145
Pepinsky, A. 386
Pepinsky, R. 544
Pérès, J. 431
Pernot, F. E. 201
Peters, Hanna 203
Peters, J. T. 43, 86, 88, 136, 143–144, 146, 168, 172, 176, 199, 203, 205, 297–298, 315, 336, 339, 358, 498, 515–517
Peterson, R. P. 386
Petit, W. H. 386
Petit-Bois, G. 482
Pettit, J. T. 178, 199
Petty, C. M. 386
Pfeiffer, J. 490
Philips, N. V. 481, 524
Phillips, R. S. 188, 386
Piatt, D. R. 149
Picard, É. 81, 306
Pickering, W. H. 386
Picone, M. 356
Pidduck, F. B. 238, 258, 499
Pierce, F. 386
Pierce, T. A. 531
Piper, E. M. 386
Pipping, N. 96, 519
Pitiscus, B. 89, 371, 390–398, 498, 515–516, 552, 556–560, 562
Pitt, G. J. 412
Placzek, G. 108
Plana, J. 235, 258, 277–278, 314
Planck, M. K. E. L. 175–176
Plesset, M. S. 386
Plummer, H. C. 249
Pluvinage, P. 369
Poehlmann, Jeanne 386
Poetius, J. M. 172
Poggendorff, J. C. 86, 226, 449, 559
Poincaré, H. 426
Poiseuille, J. L. M. 437
Poisson, S. D. 61, 99, 179–180, 301, 540
Polachek, H. 386
Poleni, G. 136
Poletti, L. 532
Pollard, A. W. 397
Pollard, W. G. 188
Pólya, G. 399–400
Pomerene, J. H. 378, 440
Poole, E. G. C. 184
Pooler, L. G. 496
Popovich, M. 386
Porath, F. R. 386
Poritsky, H. 545
Porri, 552, 560
Porter, R. J. 43
Postel, E. E. 386
Postley, J. A. 386
Potin, L. 89, 223, 225, 233, 235–236, 241, 255, 258, 268, 275, 278, 307, 314, 449
Potter, A. 34
Poulet, P. 120
Powell, E. O. 477, 485
Powers, R. E. 496
Prătorius, J. 554, 560
Prager, W. 93
Present, R. D. 188
Price, B. 335
Pringle, R. W. 61
Prokeš, A. 20, 40

Proshko, V. M. 299
Prowe, L. 560
Prytz, H. 138
Ptolemy, 560, 562
Puckett, W. T., Jr. 386
Purdie, R. H. 389

Quade, E. S. 387
Quastinsky, E. S. 387

Raab, S. 237, 258
Rademacher, H. 131, 380, 382, 387, 437
Ragot, C. L. 84
Rahmel, H. 380
Rahn, J. H. 172
Raiford, T. E. 412
Rajchman, J. A. 133, 435–436, 444, 544
Rajna, M. 20, 40
Ramanujan, S. 23, 177, 298, 468, 550
Ramus, P. 559
Raphson, J. 132
Rasmussen, C. 387
Rawlins, R. E. 387
Ray, B. S. 27, 29
Rayleigh, J. W. Strutt, *Baron* 238, 258–259, 335, 338, 450, 483, 495–496
Raytheon Manuf. Co. 286, 295, 317, 327, 382, 446, 540
Rea, E. A. 387
Read, D. N. 330
Reck, M. 137
Redgrave, G. R. 397
Redheffer, R. 389
Rees, Mina 133, 380, 544
Reeves Instr. Corp. 326–327, 501
Regiomontanus, 553, 562
Reid, W. T. 387
Reinhardt, F. 493
Reinhold, E. 553
Reiss, S. 522
Reiz, A. 26, 40, 416, 473
Renner, H. W. 60
Reno, F. V. 242, 246, 258
Renwick, W. 490
Reynolds, W. A. 60, 199, 437
Rheticus, G. J. 296–297, 390, 394–398, 515–516, 531, 552–560, 562
Rhodes, *Mrs.* Ida 134, 380, 382, 387
Rice, S. O. 307–308
Rich, K. C. 387
Richards, H. W. 548
Richardson, H. 389
Riche de Prony, G. C. F. M. 394–396, 559
Richmond, D. E. 387
Richmond, H. W. 95, 423
Richter, G. F. 172
Ridenour, L. N. 387
Riemann, G. F. B. 366, 475–476, 546
Riordan, J. 179
Ritson, D. M. 415
Rivard, D. F. 143
Robbins, C. I. 414
Robbins, F. 234, 237, 249, 254, 258, 336
Robbins, L. 387
Robbins, W. 387
Robert, H. C., Jr. 23
Roberts, R. C. 199
Robinson, C. 340
Robinson, G. 98, 116, 419, 468
Rock, Sibyl M. 61, 387
Rodes, H. P. 387
Rodgers, L. M. 102
Rodrigues, *see* Azeredo Rodrigues
Röntgen, W. C. 259
Rogers, S. 387
Rogers, T. A. 387
Rohrer, J. 88
Rolle, M. 329, 469
Romanus, A., *see* Roomen, A. van
Romig, H. J. 179
Roney, R. K. 387
Room, T. G. 544
Roomen, A. van 559
Rosa, E. B. 232–234, 238–239, 251, 253, 256, 258–259
Rose, M. E. 545
Rosen, E. 553, 560
Rosen, S. 387
Rosenbach, J. B. 232, 242, 259, 278, 314
Rosenblueth, A. S. 543
Rosenhead, L. 31, 61, 108, 119, 203, 223, 229–230, 237, 240, 245, 259, 276–277, 305, 314, 371, 450, 495, 531, 551
Rosenthal, 171
Rosenthal, D. 387
Ross, C. 545
Rosser, J. B. 474
RSMTC (R. Soc., Math. Tables Comm.), 333, 337, 339
Rubbert, F. K. 447
Rubinoff, M. 438
Runge, C. 443
Runkle, J. D. 237, 259, 278, 314
Russell, A. 232–233, 239, 259
Russell, H. N. 113–114, 192, 195, 199
Russell, J. B. 133, 365
Russell, W. T. 387
Russo, F. 212
Rust, A. 420
Rutland, D. 387

Ryan, E. F. 387
Ryayec, E. A. 387
Rybner, J. 173–174, 199, 516–517
Rydberg, J. R. 330
Rylands, T. G. 498

Sachs, A. J. 169–170
Sadler, D. H. 31–32, 34, 110, 112, 141, 204, 310, 333, 336–337, 339, 362, 531, 548
St. Hilaire, M. 309
St. John, E. E. 387
Saint-Venant, B. de 483
Sainte-Lagüe, A. 514
Sakai, A. 532
Sakamoto, S. 223, 225
Sakuraba, S. 107
Sakurai, S. 231–232, 234–236, 238, 247, 257, 259, 276, 314
Salamandra, G. D. 109
Salamonovich, G. M. 387
Salin, J. 203
Salvosa, L. R. 316
Salzer, H. E. 18, 41, 64, 97, 107–108, 119–121, 169, 187, 205, 316–317, 358, 414, 424, 472, 515–516, 519, 532
Samoĭlova-Ĭakhontova, *Miss* N. S. 89, 232, 234–235, 241–242, 259, 264, 266, 272, 276, 278, 307, 314
Sampson, J. 498
Sampson, R. A. 392, 397
Sandeman, I. 101
Sandon, F. 333, 338
Sang, E. 143
Sarahan, B. L. 387
Sarrus, P. F. 518
Sarton, G. 563
Sassenfeld, H. M. 463
Sasuly, M. 300
Sauer, R. 197–198
Saunders, F. 387
Savage, *Mrs.* Grace O. 498, 559
Savage, L. J. 547
Savedoff, Lydia 544
Savidge, H. G. 335
Savidge, W. L. 25
Sawyer, W. W. 399–400, 473
Saxon, D. S. 387
Scarborough, J. B. 468
Schady, 22
Schaeffer, A. C. 387, 546
Schelkunoff, S. A. 306, 387, 482
Schell, E. 544
Schendel, J. W. 387
Scherer, 136
Schering, E. J. 252
Schey, O. 387
Schiffer, M. 546
Schlömilch, O. X. 103, 105, 233, 235, 253, 259, 279, 281, 314
Schmidt, J. G. 232, 241–242, 259, 266, 279, 281
Schmutzler, A. E. 389
Schöner, J. 553
Schomaker, V. 60
Schomann, 355
Schott, G. A. 549
Schuck, O. H. 134
Schürba, W. 146, 452
Schultz, O. T. 240, 258–259
Schulze, F. 19, 40
Schunman, G. A. 387
Schutz, B. 387
Schwarz, H. A. 103, 234, 247, 259–260
Schwarzschild, M. 160–161
Schwinger, J. 524
Scoville, R. R. 387
Scultetus, A. 390
Sédillot, J. J. 226
Seeber, R. R. 212–213, 327–328, 382, 387, 481
Seeger, R. J. 436
Seewald, F. 472
Segal, B. I. 299
Segner, J. A. 226
Segner, J. W. 226
Seidel, P. L. 221, 350
Seidel, W. 387
Seiden, Esther 373
Seifert, H. S. 387
Séligmann-Lui, G. 256
Sepmeyer, L. W. 387
Seth, B. R. 483
Shaffer, P. A., Jr. 60, 387
Shannon, C. E. 491–492
Shapley, H. 113–114, 192–193, 199
Sharp, A. 143
Sharpe, H. D. 61
Sharpless, T. K. 57, 130, 327, 379–380, 434, 444–445, 540, 542
Shatrovskiĭ, L. I. 299
Shea, J. D. 300
Sheehy, M. J. 387
Sheppard, C. B. 130–131, 143, 438, 444, 470
Sheppard, W. F. 143, 180, 337, 339
Sherman, Iva 352
Sherman, L. 387
Sherwin, H. 562
Sherwood, T. K. 240, 259
Shiras, R. N. 387
Shook, C. A. 237, 241, 251, 259

Shoor, B. A. 387, 547
Shor, W. 547
Shortley, G. 350
Sidgwick, *Mrs.* H. 238, 258–259
Siegel, Roselyn 387, 545
Silberstein, L. 232, 235, 242, 259
Silverman, L. L. 387
Simon, L. E. 133
Simpson, T. 470
Simpson, T. W. 494
Singer, T. 356, 371, 481
Sink, R. F. 387
Sitterly, B. W. 195
Skinner, *Miss* P. M. (*Mrs.* Truscott) 108, 116
Skogland, J. F. 176
Skogstad, Anna L. 387
Slosse, E. A. 21, 88–89
Slutz, R. J. 378, 440, 443, 541, 545
Sluyter, L. J. 387
Small, A. M. 387
Smekal, A. 240–241, 259
Smiley, C. H. 34, 36, 59, 112, 115–116, 139, 145, 196–197, 311, 316, 371, 420, 422, 529, 531
Smirnoff, M. V. 387
Smirnov, N. V. 520
Smith, B. A. 25, 62
Smith, C. V. L. 380, 387
Smith, D. B. 102
Smith, E. R. 26, 416, 473
Smith, G. L. 387
Smith, H. J. S. 92, 334
Smith, L. B. 18–19
Smith, O. K. 314, 387
Smith, R. 549
Smith, R. C. T. 414
Smithson, J. L. M. 245, 254, 260, 516
Smoliar, G. 491
Snedecor, G. W. 363–364
Snell, W. 451, 561–562
Snow, C. 176
Snyder, Frances E. 350, 373
Snyder, R. L. 327, 382, 387, 440, 489
S.A.E.W.E.B. (Soc. Automotive Engins., War Engin. Bd.), 232–233, 247, 259
Sørensen, K. S. 516
Sokolnikoff, E. S. 539
Sokolnikoff, I. S. 387, 539, 545
Soldner, J. 308, 314
Sollins, A. D. 205
Sommer, H. H. 387
Sonin, N. 97, 147
Sorgenfrey, R. H. 387
Soroka, W. W. 448
Sosbee, H. F. 387
Souders, M. 387
Southwell, R. V. 57, 132, 546
Sowers, N. E. 387
Sparks, C. M. 238, 251, 259
Sparrow, C. M. 479
Speidell, J. 371
Speiser, A. 170–171
Spence, W. 42, 226, 477
Spenceley, G. W. 89–91, 205, 232, 234, 236, 245–246, 260, 262–263, 266, 269, 275, 279–280, 314, 334, 516
Spenceley, Rheba M. 89–91, 232, 234, 236, 245–246, 260, 262–263, 266, 269, 275, 279–280, 314, 334, 516
Spencer, D. C. 546
Spielrein, J. 237, 239, 260
Spinrad, B. I. 545, 547
Sprague, R. E. 387
Stamford, F. K. 171
Stankiewicz, Lidia 479
Starr, C. 387
Staton, J. 483
Stearns, E. V. B. 387
Steck, M. 145
Steed, D. V. 387
Steele, F. 387
Steffensen, J. F. 407, 468
Stegemann, M. 255
Stein, G. 545
Stein, J. 43, 168, 315, 358
Stein, M. L. 387
Steinbrenner, G. 89, 136
Steiner, O. T. 542
Steinhaus, H. 407
Steinitz, W. 141
Steklov, V. A. 251, 299
Stelson, H. E. 412
Sterneck, R. D. 24
Stevens, J. 144
Stevens, W. L. 333, 337, 359
Stevenson, A. C. 483
Stevenson, C. H. 387
Stevin, S. 143–144, 224–225, 451
Stewart, J. Q. 170
Stibitz, G. R. 1–2, 54, 127–128, 133, 221, 329, 376, 380, 433–434, 446, 491–492, 544
Stickney, M. E. 387
Stieltjes, T. J. 42, 143, 147, 244
Stine, V. 387
Stirling, J. 463, 470, 551
Stoker, R. L. 387
Stokes, G. G. 37, 334, 366, 437, 514
Stone, A. H. 197–198

Stone, C. 30
Stracke, G. 419
Strain, D. C. 387
Stratton, J. A. 101, 481, 526
Street, K. E. 387
Strohm, R. F. 447
Strutt, J. W. *see* Rayleigh
Strutt, M. J. O. 100–101
Struve, K. H. 66, 103, 340, 365
Student, pseud., see Gossett, W. S.
Sturm, R. 487
Sugar, A. C. 387
Sukhatme, P. V. 24
Sullivan, R. J. 387
Summers, R. 387
Suter, H. 227
Suthann, R. A. 387
Svoboda, A. 447
Swanson, Margaret D. 387
Swanson, N. M. 387
Sweeney, D. 186, 199
Swift, I. H. 387
Swift, J. D. 387
Swinford, L. H. 529, 546
Sylvester, J. J. 335, 549
Szász, O. 64, 387
Szegö, G. 416, 545–546

Tabor, L. P. 433, 489
Tai, C. T. 481
Talbot, A. N. 452
Tallqvist, A. H. H. 143, 186, 244, 260, 521
Tannery, J. 234, 247, 257, 260
Tasker, H. G. 387
Tawney, G. L. 389
Taylor, B. 181, 405–407
Taylor, E. K. 135
Taylor, G. I. 37–38, 40, 106–107
Taylor, H. 224–225
Taylor, M. 89
Taylor, R. 133
Taylor, T. T. 387
Taylor, W. J. 63
Tea, P. L. 390
Terman, F. E. 238, 260
Ternouth, E. J. 337, 339–340
Thébault, V. 22
Theodorsen, T. 545
Thiele, T. N. 470
Thoman, F. 261, 275
Thomas, D. E. 136, 308
Thomas, L. H. 544
Thomas, T. Y. 387
Thompson, A. J. 67, 143, 333, 336–340, 366
Thompson, Catherine M. 360
Thompson, J. S. 387
Thompson, P. E. 387
Thomson, J. J. 256
Thomson, W. *see* Kelvin
Thorne, C. J. 387
Tietze, H. 308
Tiller, F. M. 330
Tillitt, H. E. 387
Tillotson, G. 552
Tillotson, Kathleen 551–552
Tippett, L. H. C. 426
Tishman, Rose 387
Tisserand, F. 234
Titchmarsh, E. C. 182, 366, 475–476
Titus, J. 388
Tjänste, 213
Toben, G. 388
Tocher, J. F. 336
Todd, C. J. 388
Todd, J. 57–58, 64, 205, 215–216, 227, 333, 338, 382, 388, 451
Todd, Olga T. 388, 399
Tölke, F. 25
Tolstov, Ĭu. G. 61
Tolve, L. A. 388
Tompkins, C. B. 133, 380, 382, 388, 544
Toops, H. A. 135
Topping, J. 25
Torrès Quevedo, L. 136
Tramontini, V. N. 388
Traub, E. H. 102, 332
Travis, I. A. 129, 138, 489
Traxler, A. E. 136
Tricomi, F. 365
Tropfke, J. 449, 558
Trowbridge, W. P. 62
Truell, R. 305
Truran, J. W. J. 388
Truscott, *Mrs. see* Skinner, *Miss* P. M.
Tschunko, H. F. A. 463
T̆sesevich, V. P. 190–195, 199
Tsien, Hsue-Shen 197–198
Tsu Ch'ung-Chih 560
T̆sukhanova, O. A. 109
Tucker, L. 135
Tuckerman, L. B., Jr. 488
Tudor, Mary J. 388
Tukey, J. W. 415, 547
Tumbleson, R. C. 213
Turing, A. M. 57, 213–214, 216, 462
Turnbull, H. W. 224
Turner, H. H. 143
Tuttle, K. B. 388
Tyler, A. W. 388, 436, 441, 444
Tyler, E. F. 388

Tyler, G. W. 388
Tyndall, J. 483
Tysser, *Mrs.* R. St. H. (*later Dr.* R. O. Cashen) 333, 337

Ugarov, V. A. 369, 371
Uhler, H. S. 42–43, 143, 315, 355, 371, 398
Ulam, S. M. 382, 388, 546
Ulman, J. N., Jr. 489
U. S., Air Force 445
U. S., Atomic Energy Commission 186, 188
U. S., Bureau of Labor Statistics 161
U. S., Coast and Geodetic Survey 424, 485
U. S., Hydrographic Office 31, 33–36, 40, 43–44, 139–141, 309–311, 315–316, 529
U. S., National Advisory Committee for Aeronautics 199, 522, 524, 526
U. S., Navy, Naval Research Lab. 308, 417, 423, 446
U. S., Navy, Office of Research and Inventions 107, 116
U. S. S. R., Hydrographic Administration 420
Urata, S. 532
Ursinus, B. 89

Vahlen, K. T. 562
Vajda, S. 138
Valentine, F. A. 388
Van de Hulst, H. C. 526
Van der Pol, B. 108
Vandrey, F. 365
VanDyke, H. A. 388
Van Orstrand, C. E. 200–201, 223, 225, 371
van Veen, S. C. 230, 416
Van Vliet, C. J. 388
Van Wynen, K. G. 175
Vand, V. 548
Vanes, Olive E. 230
Vassal, V. 223, 225
Vavilov, S. 65
Vazsonyi, A. 388, 545
Vega, G. 136, 145, 396, 467, 561, 563
Velander, E. 492, 544
Verhulst, P. F. 235, 260, 266, 277, 280–281, 314
Vernon, R. 388
Verzuh, F. M. 380, 458, 462, 486
Viète, F. 143, 394, 515, 560, 562
Villey, J. 398
Vincent, G. K. 340
Viola, T. 244, 251, 260
Vlacq, A. 562
Voronoï, G. F. 531

Waddell, B. L. 388
Waddington, L. E. 134
Wald, A. 98, 524
Walfisz, A. 24
Walford, C. 67
Walker, R. M. 491
Wallace, W. P. 388
Wallis, J. 172
Wallis, W. A. 547
Walsh, J. E. 296, 388
Walters, S. S. 388
Walther, O. A. 97, 119, 137
Ward, L. E. 388
Ward, Mary C. 388
Ware, W. H. 378, 440
Waring, E. 423
Warren, C. E. 512
Warren, J. E. 388
Warschawski, S. E. 545
Washburn, H. W. 61
Waterman, A. T. 381, 388, 439
Watson, G. N. 66, 90, 235, 247, 298, 336, 352, 468, 479, oppo. 501, 531, 549–550
Watson, T. J. 135, 145, 149, 161, 216–217, 491
Wayland, J. H. 388
Wayman, A. 388
Wayne, A. 233, 260, 281, 314
Weaver, J. H. 388
Weber, 482
Weber, E. 546
Weber, H. F. 103, 336, 365–366, 499
Weber, H. M. 499
Webster, A. G. 25, 335
Webster, J. W. 388
Weems, P. V. H. 36, 109–110, 221
Weidler, J. F. 391
Weierstrass, K. T. W. 104, 223, 229, 234, 247, 249–250, 253–254, 259–260, 365, 449, 482, 563
Weigand, L. 308, 314
Weinberg, A. M. 187, 544
Weinberg, J. W. 300
Weinstein, A. 545
Weinstein, B. 256
Weinstein, J. 388
Weissenstein, H. G. 138
Welin, E. 323
Weller, R. 350
Wellmann, V. 237, 260
Welmers, E. T. 388
Wend, A. G. M. 192, 195
Wenick, R. L. 388
Wermer, J. 522
Werner, J. 553, 560

Westberg, R. 472
Western, D. W. 30, 178
Westhafer, R. L. 388
Weyl, H. 399–400
Wheatstone, C. 137
Whelan, D. E., Jr. 388
Whinnery, J. R. 388
Whipple, F. L. 419
White, W. B. 388
Whitman, E. A. 232, 242, 259, 278, 314
Whittaker, E. T. 90, 98, 104, 116, 235, 247, 419, 468
Whittaker, T. 67
Whitwell, T. 336
Widder, D. V. 545
Widell, G. A. 492
Widemnaun, J. A. 388
Wiener, N. 543–544
Wieselman, I. L. 388
Wigner, E. P. 187
Wijngaarden, A. van 21
Wilbur, J. B. 137
Wilcox, D. E. 61
Wilkas, B. C. 390
Wilkes, M. V. 214–215, 333, 338, 490, 542
Wilkinson, J. H. 132, 462, 542
Wilks, S. S. 316, 364
Willcox, W. F. 62
Willers, F. A. 493
Williams, D. 499
Williams, F. C. 57, 214, 445, 491–492, 542
Williams, K. P. 237, 260
Williams, S. B. 2, 54, 433, 443,
Wilsing, W. 388
Wilson, J. 355, 518
Wilson, O. B. 388
Wilson, R. R. 547
Wilton, J. R. 245, 260
Wilts, C. H. 512
Wingquist, E. 61
Winston, C. 416
Winter, Edna B. 388
Wishart, J. 333, 336–339
Witkowski, A. W. 233, 260
Witt, G. 233, 237, 248, 260, 272, 307
Wittke, H. 390, 467, 548
Wittstein, T. 298
Wölffing, E. 86, 449
Wolf, R. 554, 560
Wolfram, J. 171–172
Womersley, J. R. 333–334, 338
Woo, Mary 373
Wood, H. A. 388
Wood, W. L. 388
Woodall, H. J. 357
Woodbury, W. W. 388
Woods, B. M. 201, 388
Woodward, P. M. 415, 418
Wrench, J. W., Jr. 18–19, 42, 66, 120, 203, 262, 314–315, 423–424, 551, 562
Wright, E. 223–225, 372
Wright, E. M. 314
Wright, Frances W. 44
Wright, S. 225, 372
Wrinch, Dorothy M. 336
Wroński *see* Hoëne-Wroński
Würschmidt, J. 560
Wundheiler, A. W. 434
Wyatt, Willa A. 212
Wylie, P. E. 388
Wyreen, J. 388

Yamauti, Z. 176
Yarden, D. 96, 119, 299, 314, 519
Yates, F. 300, 359–360, 362, 515
Yoder, R. M. 388
Yonemura, 195
Young, D. M. 546
Young, W. E. 388
Yowell, E. C. 388
Ypei, N. 331
Yuan, S. 369
Yuster, S. T. 388

Zagor, H. I. 326–327, 382, 388
Zarqāla *see* Ibn al-Zarqāla
Zartner, H. A. 388
Zavrotsky, A. 97
Zehan, F. W. 388
Zeilon, N. 492
Ziaud-Din, M. 24, 40
Zimmermann, H. 136
Zimmermann, L. 136, 412, 423
Zinger, N. 21
Zinner, E. 560
Zizicas, G. A. 388
Zubkov, P. I. 417

203,

CLASSIFICATION OF TABLES

A. Arithmetical Tables. Mathematical Constants

B. Powers

C. Logarithms

D. Circular Functions

E. Hyperbolic and Exponential Functions

F. Theory of Numbers

G. Higher Algebra

H. Numerical Solution of Equations

J. Summation of Series

I. Finite Differences. Interpolation

K. Statistics

L. Higher Mathematical Functions

M. Integrals

N. Interest and Investment

O. Actuarial Science

P. Engineering

Q. Astronomy

R. Geodesy

S. Physics, Geophysics, Crystallography

T. Chemistry

U. Navigation

V. Aerodynamics, Hydrodynamics, Ballistics

Z. Calculating Machines and Mechanical Computation

CONTENTS

OCTOBER 1949

The California Institute of Technology Electric Analogue Computer.. 501

Recent Mathematical Tables..... 513

658 (Italy, Istituto Idrografico d. r. Marina), **659** (Dale), **660** (Sainte-Lagüe), **661** (Danjon), **662** (NBSCL), **663** (Huskey), **664** (Rybner & Sørensen), **665** (Harvard Univ., Comput. Lab.), **666–667** (Lehmer), **668** (Pipping), **669** (Yarden & Katz), **670** (Salzer), **671** (Jeffreys), **672** (Smirnov), **673** (Marcum), **674** (Grover), **675** (Hartree & Johnston), **676** (Huckel), **677** (Lowan & Horenstein), **678** (Magnus & Oberhettinger), **679** (NBSCL), **680** (Bouwkamp), **681** (Levine & Schwinger), **682** (Meixner), **683** (Hallén), **684** (Le Caine), **685** (Van de Hulst), **686** (Financial Publ. Co.), **687** (Italy, Istituto Geografico Militare), **688** (Hoehne), **689** (Azeredo Rodrigues), **690** (Sadler)

Mathematical Tables—Errata..... 531

160 (D. H. Lehmer), **161** (D. N. Lehmer)

Unpublished Mathematical Tables..... 532

82 (Poletti), **83** (Salzer)

Automatic Computing Machinery..... 532

Discussions (Procedure for the machine or numerical solution of ordinary linear differential equations for two-point linear boundary values); Bibliography, Z–IX (**1–4.** Anon., **5.** Arenberg, **6.** Booth-royd & Cherry, **7.** Brainerd & Sharpless, **8.** Bronzo & Cohen, **9.** Chu, **10.** Davis, **11.** Goldstine & Neumann, **12.** Hartree, **13.** McCool, **14.** Sharpless, **15.** Wiener); News

Other Aids to Computation..... 548

Bibliography Z–IX (**1.** Brown, **2.** Carter & Sadler, **3.** Knauss, **4.** Richards & Sadler, **5.** Vand, **6.** Wittke)

Notes..... 549

106 George Neville Watson—Table Maker; **107** The Lemniscate Constant; **108** Newman and George Eliot; **109** Rheticus, with Special Reference to his Opus Palatinum

Query..... 561

32 French and Russian Translations of a Vega-Bremiker Table

Queries—Replies..... 561

41 Girard and Snell Tables; **42** Pitiscus Tables

Corrigenda..... 563

Indices..... 564

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01
13
531
532
532
548
549
561
561
563
564